A GENERAL APPROACH TO THE CALCULATION OF RELIABILITY INDICES FOR AN ELECTRIC POWER TRANSMISSION SYSTEM

A THESIS

Presented to

The Faculty of the Division of Graduate Studies

By

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

in the School of Electrical Engineering

Georgia Institute of Technology

September 1976

A GENERAL APPROACH TO THE CALCULATION

OF RELIABILITY INDICES FOR AN ELECTRIC

POWER TRANSMISSION SYSTEM

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ACKNOWLEDGEMENTS

I wish to express my appreciation to Dr. Roger P. Webb, my thesis advisor, for his sustained support, encouragement, and constant motivation throughout the development of this dissertation. I wish to thank Mr. John Day, of the Westinghouse Electric Corporation, who has served as a member of the reading committee, but who has also served as a valuable contact with the power industry. Appreciation is also extended to Drs. Atif S. Debs and Edward W. Kamen who have served as reading committee members.

Mrs. Kathy Massett worked many late hours typing this manuscript and has my lasting gratitude for her patience which enabled me to complete this work in the time allowed.

My deepest appreciation goes to my parents and especially to my wife, Julie, whose loyal support and encouragement made it possible for me to pursue this work.

I would like to acknowledge the Westinghouse Electric Corporation for the financial support necessary to conduct this research.

I would also like to thank Georgia Power Company for providing network data and system reliability information.

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SUMMARY

This dissertation considers the problem of evaluating the anticipated performance of an electric power transmission system. Specifically, a computational procedure is presented which calculates reliability indices for a network operating with uncertainty of component availability and load. The load and network are modeled as stochastic processes. Indices are calculated which relate to the event that the load will not be satisfied; the frequency of this event for each bus in the network, the expected portion of time associated with this event.

In order to evaluate the performance of the network under various outage conditions, the a.c. load flow equations are solved. A method of decoupling the real and reactive power flow equation is used for this solution. The uncertainty in load is analyzed by linearizing the a.c. load flow equations about the mean load.

The effect of overloaded lines is analyzed with the use of sensitivity factors. These sensitivity factors are calculated from the decoupled reactive power flow equations and represent the change in line flow in a line due to a change in power flow at a bus. These sensitivity factors eliminate the need for repeated solutions of the load flow equations each time an overloaded line is located.

In order to evaluate this procedure, the results of the procedure are compared with a Monte Carlo simulation of a 90-bus network. The procedure is also compared to previous conditional probability methods on three- and five bus networks.

The major contribution of this research is the use of sensitivity analysis and stochastic load flow techniques to permit the calculation of reliability indices for a large network. The procedure is designed to be efficient with regard to computer time while providing sufficient detail and accuracy for the indices to be meaningful. An additional attribute is that the procedure is useful using only readily available data.

CHAPTER I

INTRODUCTION

Motivation

Since the early beginnings of electric power transmission, the load placed on the transmission system has consistently grown. Because of this increased demand, electric utility companies are continually planning and building additions to the power transmission network. This thesis develops a computational process which is useful for this planning process.

Historically, transmission planners have simply added more lines to the network as the demand for more and better service increased. However, for economic and environmental reasons, the practice of simply adding lines to alleviate potential overloads results in an overly conservative network design and is no longer an acceptable solution to the problem of satisfying a growing demand. The transmission planner must have a tool to evaluate the impact of proposed network changes before construction actually takes place. The same economic factors that preclude the continual adding of lines also require that this procedure be efficient, as many possible network changes must be evaluated before each network expansion.

The ability of the power transmission system to transport electric power from the points of generation to the points of demand in the quantities and at the times required is defined as the "reliability" of the system. If a system can satisfy all the requirements of the demand all the time, it is completely reliable. Conversely, if a system never satisfies the demand, it is completely unreliable. Since no system is completely reliable or completely unreliable, a method of quantitatively assessing the reliability of a system is needed.

The Problem

Since no transmission network can completely satisfy all the demand all the time, a set of indices is required which gives a quantitative indication of how well the network satisfies the demand. This measure is a set of indices rather than a single index since the various loads are not satisfied equally. This disparity of service is caused primarily by the geographic nature of the problem. That is, load points which are located near points of generation generally receive a higher quality of service than points far removed from generation. Additional causes of the disparity are the varying requirements of the load points. That is, an industrial customer whose load requires a complicated start-up procedure after a power failure is interested in the number of service interruptions, whereas a residential customer is basically interested in total time of service interruption.

One use of the reliability indices is as a tool for transmission planners in cost-vs-reliability studies. These studies are useful as a direct planning aid and also can be used as inputs to rate allocation proceedings. The procedure for transmission system planning utilizing the reliability measures developed in this thesis is a five step process. First, reliability indices are calculated for the existing system.

Second, both engineering and managerial judgement must be used to determine whether or not the existing system will perform acceptably for the planning period. If the existing facilities are adequate, the planning process is complete. Step three is to examine the reliability indices in detail to determine where the weaknesses are and propose specific additions to the network. Fourth, reliability indices for the proposed network are calculated. Fifth, these indices are used to determine the effect of the network addition.

Two reliability indices are calculated for each load point in the system:

$$p_k = \text{probability of service interruption at bus k}$$

$$= \frac{\text{expected value of total duration of service interruption}}{\text{total time}} \quad (1)$$

$$f_k = \text{expected frequency of service interruption}$$

$$= \frac{\text{expected number of instances of service interruption}}{\text{total time}} \quad (2)$$

where k is the bus number of the customer of interest.

Both indices are necessary to provide an accurate indication of the quality of service being provided. The frequency index is necessary because any interruption of service constitutes an inconvenience and/or expense for customers, especially industrial customers. However, interruptions of short duration are more desirable than interruptions of long duration. Therefore there is also a need for the duration index.

The term "service interruption" refers to the condition that a demand exists at the bus and the demand cannot be satisfied because of overloaded network elements or low bus voltage. Both indices involve computation of expected values since the service interruption is a function of network and load variables which are modeled as stochastic processes.

Background

The current techniques in the field of transmission system reliability evaluation have their origins in the literature of 1964 when three independent papers[1-3] appeared which presented three different approaches to the problem. The first of these papers[1] proposed a Monte Carlo study of the problem. The procedure models the network as a stationary, first-order Markov process. Each network element is modeled as a two-state Markov process with exponentially distributed time to transition. The network states are described by the combined states of the network elements, that is, the state vector of the system. This model is the basic model used for most current techniques. Two undesirable characteristics of the Monte Carlo type of study are: (1) in order to study a complex network, an unacceptably large amount of computer time is required; (2) line failure producing catastrophic results may go undetected.

Another 1964 paper[2] presented a more efficient, but less accurate, algorithm. This method calculates in an approximate manner the probability of an outage longer than T minutes occurring during any given day. This method constructs, for each bus, a chart of the probability of outage longer than T minutes for various values of T. Information about the frequency and duration of service interruptions can be extrapolated from these tables. This extrapolation is cumbersome and involves "seat-of-the-pants" calculations rather than precise quantitative calculations. This paper provides the origin of the current practice of calculating expectation of both frequency and duration of outages.

The third 1964 paper[3] presented on approximate method of dealing with simple series-parallel network configurations under the influence of storm-related outages. A computer program to perform the calculations appeared a year later.[4] This method has been extended to determine reliability of supply to several stations with multiple paths. This extension included, in an approximate manner, supply interruption caused by circuit overloads and scheduled outages. Recently, the method was extended again to consider three parallel lines[6] and to include different modes of load point failure.[7] The later work also allowed for a three state weather model. However, in all forms discussed above, this method cannot be applied to a complex network.

Billinton and Bollinger[8] used the Markov model of DeSieno and Stine[1] and incorporated the two state weather model of Gaver, et al.[3] Accurate Markov calculations were presented for two and three lines in parallel.

For this model, the network is described by the states of the network elements in conjunction with the weather state, i.e. normal or stormy. This model contains twice as many states as the basic Markov model, but allows for component failure rates to be altered during stormy weather. The two-state weather model has been used successfully in modeling distribution systems where the increased component failure rates during storms increase the probability of multiple outages. However, data indicate that for transmission systems, weather produces only a localized effect. This localization results from the higher voltages and the localized nature of storms. (See the discussion of reference 10 and reference 27, pp. 194-195.) Therefore, more recent publications have abandoned the idea of two-state weather. Stanton uses this model but obtains directly the steady-state solution to the state equations.[9]

Christiaanse[10] presented a simplification to the direct Markov approach. This approach partitions the state space into compound states. This formulation led to a reduction in the number of possible states and thus a reduction in the number of equations to be solved. Ramamoorty and Balgopal[11] proposed using block diagram techniques for state reduction. The calculations required for all forms of direct application of Markov theory are prohibitively extensive for all but the simplest of networks.

The technique of using minimal tie and cut sets to identify failure modes of a transmission system was first introduced by Stanton.[9] The theorems and techniques of linear flow theory pertinent to this calculation can be found in Ford and Fulkerson,[12] Jensen and Bellmore,[13] and Nelson, Batts and Beadles.[14] Batts[15] presents a general computer program to determine reliability bounds for a network using minimal cut and tie sets. Endrenyi, et al.[16] have extended the idea of minimal cut sets to a network with switching after faults.

All the techniques presented above use the criterion of connectedness for state evaluation. This criterion overlooks the fact that all transmission system elements have finite limits on the amount of power that may be transmitted. There is also no requirement that bus voltage limits are obeyed. Because of these shortcomings, these mathematical procedures can not be considered solutions to the problem. However, these techniques lay the foundation for the models which will be used in this research.

The PERU program[17,18] which is used in Europe to study a two-area interconnection uses a Monte-Carlo simulation and a d.c. load flow calculation for evaluation of effects. The Monte-Carlo technique is inappropriate for a complex configuration for the reasons discussed earlier. The d.c. load flow calculation has questionable accuracy for this situation.

Pang and Wood[19] study an n-area interconnection using the theorems of critical minimal cuts presented by Ford and Faulkerson.[12] This method becomes too cumbersome when applied to a complex network. No provision is made to evaluate bus voltages.

Recently, a method of calculating transmission reliability has appeared in the literature which is based on calculating the conditional probability of all possible network configurations.[20-22] The equations for this method are:

p_k = probability of service interruption at bus k

$$= \sum_j [P(B_j) PL_j]; \tag{3}$$

f_k = expected frequency of service interruption at bus k

$$= \sum_j [F(B_j) PL_j] \tag{4}$$

where:

B_j	is a specific outage condition in the transmission network
$P(B_j)$	is the probability of existence of outage B_j
PL_j	is the conditional probability of load at bus k exceeding the maximum load that can be suppled at that bus during outage B_j (i.e. conditioned on outage B_j)
$F(B_j)$	is the frequency of occurrence of outage B_j
$\sum_j$	indicates summation over all outage conditions of interest.

This method is referred to in the literature as "the conditional probability approach." This method is an acceptable approach to the reliability problem. However, in this form, this method is incomplete in that there is no accurate technique for calculating the quantity PL_j. Since this is the key element in equations 3 and 4, methods of calculating PL_j are required.

Recognizing the problem of calculating PL_j, Bhavaraju and Billinton[23] proposed a method they call "the extended conditional probability approach." This method incorporates the load model used by Ringlee, _et_ _al_.[24-26] for generation reliability calculations. The load model is discussed in detail in reference 25. The load at each load point is modeled as different magnitudes of peak load occurring for a known number of days. That is, the continuous load function has been approximated by a square wave. A constant peak load is assumed to exist for a fraction of the day and then the load drops to a low value. The duration of the peak is assumed to be exponentially distributed and the bus loads are perfectly correlated, i.e. all loads peak at the same time.

The load parameters are as follows:

Peak load levels	$= L$ (MW)
Number of days L occurs in a period D	$= n_L$
Period D in days	$= \sum_L n_L$
Expected duration of peak load	$= e$(days)$(e<1)$
Probability of load L existing in period D	$= A_L = \dfrac{n_L e}{D}$
Low load level in period D	$= L_o$
Probability of low load existing in period D	$= 1-e$
Transition rate to greater load	$= \eta_{+L} = 0$
Transition rate to lesser load	$= \eta_{-L} = 1/e$
Transition rates for low load	$= \eta_{+L} = 1/(1-e)$
	$\eta_{-L} = 0.$

For each load level L, there are certain outage states for which the system will be able to supply only a part of the load due to limited line capabilities. These states can be termed negative margin states for the specific load level. A simplified expression is presented in reference 25 to obtain the cumulative frequency of negative margin states utilizing the probability and frequency of outage states. This expression is used in reference 23 to obtain the following results.

$$\text{Expectation of loss of load at a bus} = \sum_{L} n_L A_G \text{(days/period)} \tag{5}$$

$$\text{Frequency of loss of load} = [1-e]\cdot[f_{GO} + A_{GO}[\eta_{-L} - \eta_{+L}]] + \sum_{L} A_L[f_G + A_G[\eta_{-L} - \eta_{+L}]] \tag{6}$$

$$\text{(occurrences/day)}$$

where:

f_G is the cumulative frequency of outage states that result in negative margin for load level L, corrected for frequency of transfers within the states.

f_{GO} is the cumulative frequency of outage states for low load.

A_G is the cumulative probability of outage states that result in negative margin for load level L.

A_{GO} is the cumulative probability for low load.

Equations 5 and 6 require information on the maximum load that can be supplied to the load points for various outage states. It is unclear, however, how this information is to be obtained. An example is given of a simple network for which this calculation can be "eyeballed." The authors state that this information can be calculated using approximate or accurate load flow analysis. This statement is misleading in two respects. First, the load flow analysis required is not a simple load flow for a fixed load but a series of load flows for ever-increasing system loads. This in itself is a very inefficient method of searching for the answer. The second problem with the authors statement is that no method is presented for penalizing buses for overloaded lines. That is, many times when a line overloads it is unclear which load will not be satisfied.

Two recent developments in the field of load flow calculations have application to the mathematical procedures for reliability assessment. These developments are the stochastic load flow and the decoupled load flow.

The idea of performing load flow calculations for non-deterministic loads has been the basis for two recent papers.[29,30] These papers discuss the general ideas associated with the problem and a computer program is presented to perform this calculation for independent bus loads.

Stott and Alsac[31,32] have developed a more efficient algorithm for performing deterministic, moderately accurate load flow calculations.

This method decouples the equations for real and reactive power flow and introduces approximations which remove the requirement for matrix inversion in every Newton-Raphson iteration.

Approach to the Problem

For this research, the conditional probability approach is used as the basis for the calculation. However, the basic method is improved by: (1) using the faster, more efficient load flow equations presented by Stott; (2) allowing the load model to be stochastic in nature and thus allowing the peak to be unknown; (3) coupling to the stochastic load model, load flow equations which are stochastic in nature and thus removing the requirement for the successive load flow calculations; (4) implementing a line dropping algorithm to remove the triangulation required of the load flow matrices when a line is removed from service; (5) modeling the network in such a way that the localized nature of storm related failures is included; (6) modeling the generation in such a way that the geographic locations as well as the dispatching algorithm are included; and (7) using sensitivity factors to assess the effects of overloaded lines.

Included in the problem formulation are mathematical models of the pertinent elements of the system. The system model is divided into three parts: the load model, the network model, and the generation model. The load model incorporates the idea of daily load variation and allows for uncertainty in the prediction of load pattern. The network model allows for the uncertainty of failures of network elements and length of repair times. Thermal and stability limits for all elements are also included. For this procedure, available generation is specified with generation dispatching done on an economic basis. Each generator has real and reactive power limits which must not be exceeded. An additional feature of the models is that all model parameters are obtainable from existing network data.

The computational procedure includes an a.c. load flow calculation in order to determine the effects of loading on the various elements of the network. Since the system load is not known exactly, this calculation is stochastic in nature. The load flow calculations constitute the major portion of the calculations required for the procedure. Therefore, the most efficient load flow techniques available are used.

The load flow calculation identifies buses that do not meet voltage specifications and lines that are overloaded. However, it is necessary to calculate the effects of the overloaded line on the buses. Previous techniques do this by removing the overloaded line from the network and performing the load flow calculations again. This method approximates the second load flow by calculating the sensitivity of line flow to bus demand. These sensitivity factors indicate the buses that will experience service interruption due to line overloads.

CHAPTER II

PROBLEM FORMULATION

Introduction

In order to develop a procedure which will calculate network reliability, two tasks are required: (1) development of a quantitative definition of reliability; (2) mathematical modeling of the components of the system necessary to represent the operation of the network. This chapter presents the definition of reliability as used for this thesis and describes the models developed for the computational procedure.

Measure of Reliability

The term "reliability" when referring to an electric power transmission system is generally defined as the ability of the system to adequately transport electric power from the points of generation to the points of load. The term "adequately" in this context has many definitions. For this thesis, "adequacy" implies that all network elements are within specified limits. Specifically, the conditions for adequate operation are: no lines are overloaded, all buses are within voltage limits, all generators are operating within real and reactive power limits, and all loads are satisfied.

The system behavior at each bus is modeled as a two state Markov process. The two states are defined as: (1) the system is performing adequately as defined above; (2) the system is not performing adequately. The indices are defined as the probability and frequency of this process being in the inadequate state. The indices, therefore, represent estimates of the portion of time and the number of occurrences of network unreliability. Motivation for the two indices is the fact that an electric power utility has many customers with widely varying needs. For example, an industrial customer whose load requires a complicated start-up procedure following a power failure would consider a system with many service interruptions less reliable than a system with less interruptions of longer duration. Conversely, most residential customers would prefer more interruptions if the total time of interruption were less.

The indices are required to be sensitive to the network parameters which affect network reliability. For example, the indices are sensitive to changes in predicted load statistics. That is, the indices are sensitive to both the mean and the variance of the network load distribution which is assumed normal. The indices will be used to compare various network configurations and therefore were constrained to be consistent with changes in network parameters. The concept of consistency is discussed further in Chapter IV. The method was developed to be practical in that the computational procedures required by the indices are constrained to use as little computer time as possible.

The indices presented in Chapter I as equations 3 and 4 are used as the basis for the computations of this method. These equations are repeated here.

$$p_k = \text{probability of service interruption at bus } k$$

$$= \sum_j [P(B_j)PL_j]; \qquad (3)$$

$$f_k = \text{expected frequency of service interruption at bus k}$$

$$= \sum_j [F(B_j)PL_j] \quad (4)$$

where:

B_j is a specific outage condition in the transmission network

$P(B_j)$ is the probability of existence of outage B_j

PL_j is the conditional probability of load at bus k exceeding the maximum load that can be supplied at that bus during outage B_j (i.e. conditioned on outage B_j)

$F(B_j)$ is the frequency of occurrence at outage B_j

$\sum_j$ indicates summation over all outage conditions of interest.

The basis of these calculations has been presented in the literature cited in Chapter I. The assumption used for these calculations is that the network can be represented as a stationary random process, and furthermore, that this process possesses the ergodic property of equivalent time and ensemble averages. This assumption implies that the probability of an event represents an estimate of the portion of time that a particular event will exist.

The indices are bus oriented. That is, a pair of indices exist for every load bus in the network. The specific characteristics of the load specify which indices are of interest. The reliability of the network itself can then be formulated from the set of bus indices.

Models

Before any form of quantitative analysis can be performed, mathematical models must be developed. Specifically, before reliability calculations can be made on a power transmission system, the pertinent components of the system must be modeled. The single function of a power transmission network is the transportation of electric power from the points of generation to the points of load. The system, therefore, has three major components: (1) the generation; (2) the transmission network; (3) the load. Each of these components is modeled individually and the models are tied together with appropriate computational techniques.

The computational method present-d in this thesis is based on the theorems of conditional probability. It is therefore required that the models listed above be compatable with the calculations associated with the conditional probability approach. In addition, for planning purposes, it is required that the models represent the system elements at some time in the future. Since the events which will occur in the future cannot be predicted exactly, uncertainty must be included in the models. That is, load patterns and network element failures can only be predicted in an approximate or average manner. The models employed in this thesis are formulated to represent those characteristics which are unknown as stochastic. Those characteristics which are known are modeled exactly. The models, therefore, are formulated to depict as accurately as possible the characteristics of the elements of the power transmission network.

System Load Model

The system load at any given point in the future can be predicted only in an approximate or average manner. That is, although the load on a particular day in the future may not be known exactly, an extrapolation of past and current loads can predict the average load over a number of days. The model thus represents the load as a stochastic process with a known mean and variance. This allows for the uncertainty in the load but accounts for the predictability of the mean.

Another characteristic of the load which is included in the model is the daily load variation which is characteristic of all power systems. The daily load variation generally peaks at some time in the afternoon and reaches a low point during the night. This load variation is approximated by a square wave representing the peak and low loads. The peak level of the load is influenced by many unpredictable factors. The specific conditions affecting the peak must be determined for each study. The primary factor affecting the peak load is weather. Although the weather is unpredictable, it can be predicted in an average manner. For example, average temperature can be predicted as well as the approximate number of hot or warm days for some period. Since weather affects the load primarily by changes in ambient temperature, these effects can be predicted in an approximate manner.

The low load level is more predictable and represents the constant or "base" load on the system. The low load level is relatively constant from day to day.

The sequence of daily peak loads is represented as a vector, first-order Markov process. This process is depicted in Figure 1.

The practice of modeling system loads as a Markov process was first presented by Hall, Ringlee, and Wood.[24-26] Data were presented which substantiated the idea of a Markov process.[26] The model represents the daily load cycle as a sequence of peak loads L_i, each of a mean duration of e days alternating with periods averaging (1-e) days of light load L_o. The sequence of peak loads is random, however, the average number of occurrences of each peak load state during the period is assumed to be known.

The system transfers from the low load state to a peak load state once a day. Transfers from a peak load state to the low load state also occur once a day. The peak loads occur in a random sequence with each peak load state occurring on the average n_i times during an interval which is D days long. When the system is in the low load state L_o, the probability that the next peak will be L_i is α_i, where

$$\alpha_i = \frac{n_i}{D}, \quad i = 1,2, \ldots ,L$$

where:

L is the number of peak load states.

When the system is in a peak load state L_i, the probability that the next load state will be L_o is unity. The frequency of occurrence of the low load state is unity. The process is assumed to be ergodic, therefore, the probability of existence of a load state is equal to the average fraction of time spent in that state.

$$P_{Li} = \frac{n_i e}{D}, \quad i \neq 0 \qquad P_{L0} = 1-e$$

$$F_{Li} = \frac{n_i}{D}, \quad i \neq 0 \qquad F_{L0} = 1 \tag{7}$$

where:

P_{Li} is the probability that load state L_i exists. (The expected portion of total time load state L_i exists.)

F_{Li} is the frequency of occurrence of load state L_i.

The system load vector, $\overline{S} = \overline{P} + j\overline{Q}$ is defined by the system load state. That is, the system load is defined by one of the vectors:

$$\overline{S}_i = \overline{P}_i + j\overline{Q}_i = (\overline{\alpha}_1 + j\overline{\beta}_i)\rho_i, \quad i = 0.1, \ldots, L. \tag{8}$$

The vector $\overline{S}_i$ which is used to define the system load $\overline{S}$ is determined by the state of the random process described above. The constants $\overline{\alpha}_i$ and $\overline{\beta}_i$ represent load distribution factors and will generally be equal for all states.

$$\overline{\alpha}_1 = \overline{\alpha}_2 = \ldots = \overline{\alpha}_L$$

$$\overline{\beta}_1 = \overline{\beta}_2 = \ldots = \overline{\beta}_L$$

This is not a requirement of the method but a limitation imposed by available data. The individual bus loads are modeled as being perfectly correlated. That is, knowledge of the load at any bus implies knowledge of the load at all buses. This is an approximation resulting from the

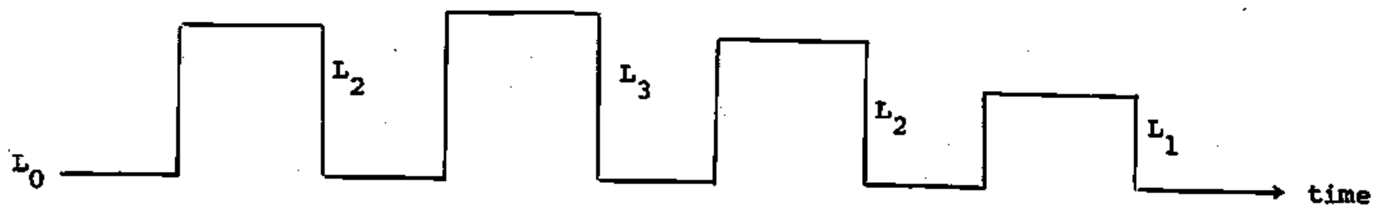

Figure 1. Load Variation With Time.

assumption that external influences will affect all loads in the same manner. In general this is not a limitation of the method since no correlation data exist for most loads.

The load, therefore, is a deterministic function of the random variable ρ_i. This random variable represents the uncertainty in the load. ρ_i is a normal random variable with mean μ_i and standard deviation σ_i. The values of μ_i and σ_i are determined by a load forecasting algorithm.

Network Model

The transmission network is a collection of elements operating together to transport electric power from points of generation to points of load. Each individual element in the network will fail at some time in the future. Generally, these networks are built with a high degree of redundancy. This redundancy is such that the loss of any single element will not normally prevent the network from satisfying the load. However, during periods of abnormally high load or scheduled maintenance. single network elements can be critical to the network operation.

Transmission line outages are caused by many different events. The primary causes of these outages are either weather related or man/machine contact. The man/machine contacts are treated as independent events. Weather effects are generally storm-related and therefore localized to a small portion of the transmission network. The independence of events as well as the localized nature of storm-related failures is included in the model.

The network is modeled as a first-order Markov process independent of the load. The network model is depicted in Figure 2. The

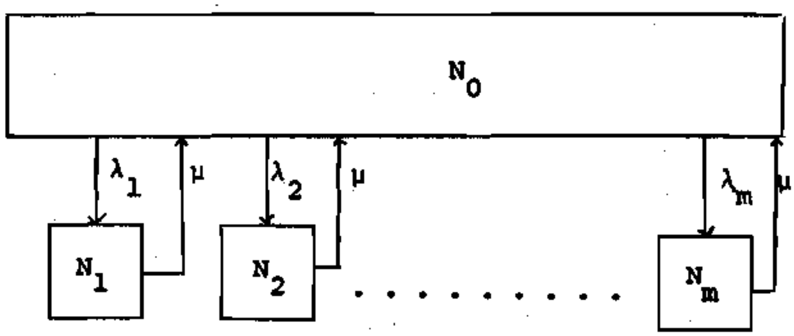

Figure 2. The Network State Diagram.

model assumes statistically independent, stationary, exponential distributions of network contingencies and repair times. The network states N_j, $j = 1,2, \ldots, m$ represent contingency states and state N_o represents the state with all components operating normally. Each contingency state is defined as a single line outage (one network elemet out of service) or a single bus outage (all elements connected to a particular bus out of service). This model allows for the independence of events causing outages by defining the contingency states to be statistically independent. The localized nature of storm-related outages is approximated by the bus outage states.

The definition of relevant contingencies in the above paragraph is the result of a study of Georgia Power line outage data for the years 1968-74. (See Table 1.) This study revealed that most multiple line outages occurred for lines connected to a common bus.

The network moves from state N_o to contingency state N_j with failure rate λ_j and from contingency state N_j to state N_o with repair rate μ. For computational purposes, the assumption is made that λ_j is a function of only line voltage and length. The repair rate is assumed to be a function of line voltage only. The failure and repair rates for buses are assumed to be the same for all buses with $\mu_{bus} = 2\mu_{line}$.

The failure rate for lines is calculated from historic outage data for all lines of similar construction in service. The failures/year-mile are calculated as the average number of failures per year-mile of operation. The mean repair rate is calculated as the inverse of the average duration of outage for all outages at the same voltage level. The failure rates for buses are calculated in the same manner.

Table 1. Summary of Georgia Power Outages Over Five Minutes.

KV	mi-yr	bus-year	no. of outages	outage/mi-yr.	total duration	average duration	no. of outages w/overlap	not. cor.	common bus	common bus/bus-yr.
230	15153	450	59	.0039	688 Hr.	11.67 Hr.	2	0	1	.0022
115	30793	1950	399	.013	1056 Hr.	2.65 Hr.	20	4	15	.0077

The calculation of λ and μ from historic data is justified by the assumption that the outage influencing factors tend to have a constant average for long periods of time. The assumption is also made that sufficient historic data is available that the failure and repair rates can be calculated to a reasonable degree of accuracy. This assumption pertaining to bus failures at a high voltage level (345 kV and above) can be shown to be incorrect. However, errors in the bus failure rates tend to be small when the errors are within the bounds considered in Chapter IV.

The following definitions pertain to the network model.

N_o	the network state with all components available
N_j, $j=1,2, \ldots ,m$	the j^{th} contingency state
λ_j	the mean failure rate from state N_o to N_j
μ	the mean repair rate
d_j	the mean duration of contingency states N_j, $j=1,2, \ldots ,m$ $= 1/\mu_j$
P_{Nj}	the probability of state N_j existing at some point in the future
F_{Nj}	the frequency of occurrence of state N_j $= \mu_j P_{Nj}$, $j=1,2, \ldots ,m$
F_{NO}	the frequency of occurrence of state N_0 $= \mu_o(1-P_{NO})$
d_0	the mean duration of state N_0 $= P_{NO}/F_{NO}$

Generation Model

Before an evaluation of a transmission network can be performed, the points of generation must be identified. In actual operation, the generation is determined by first calculating the required generation. This is generally done by adding a spinning reserve margin to the load plus the transmission losses. The spinning reserve is a safety margin determined by operating experience and set by managerial decision. After the required generation is calculated, the on-line generation is chosen from the available generation. The on-line generation is then dispatched according to an economic optimization algorithm.

The generation model used for this procedure mimics the above operation. That is, the spinning reserve is an input to the process. Once the load is determined from the load model, the load is added to the spinning reserve to determine the required generation. The losses are not included since they are not known at this point. However, the losses can be accounted for in an approximate manner by an appropriate increase in the spinning reserve specification. The on-line generation is determined by bringing generators on-line in a pre-determined order until the required generation is exceeded. The generation is then dispatched in the manner which reduces the cost function:

$$C_T = C_1 + C_2 + . . . + C_G$$

where:

G is the number of on-line generators

C_k is the fuel cost of generator i

$$= a_{0k} + a_{1k}P_{Gk} + a_{2k}P_{Gk}^2$$

P_{Gk} is the real power generated by generator k.

This is commonly referred to as "economic dispatch using a quadratic cost function." The factors a_{0k}, a_{1k}, a_{2k} are determined by the incremental heat rates and fuel costs for the individual generators. P_{Gk} is constrained to be within the thermal and stability limits for each machine. These limits are specified as maximum and minimum real power which can be generated by each machine.

In addition to the real power limits, each machine also is constrained by reactive power limits. The reactive power generated by each machine is determined as that reactive power required to maintain some constant output voltage level. If the reactive power required exceeds either the maximum or the minimum reactive power limit, the reactive power is set to that limit and the output voltage allowed to fluctuate.

The generation model is included for the sole purpose of defining the points of generation. Although generators generally fail more frequently than transmission lines, the generators are modeled as always available. This feature of the model is included for two reasons: (1) A stochastic generation model would increase the complexity of the computational procedures; (2) The unreliability of the generators would obscure the unreliability of the transmission network. Since methods of calculating generation reliability currently exist, the separation of transmission reliability from generation reliability is desirable.

CHAPTER III

COMPUTATIONAL PROCEDURE

Introduction

Chapter II presents the reliability indices which have been defined for this thesis. Mathematical models are also presented which facilitate calculation of these indices. For the indices to be practical, a computational procedure is required which will calculate the indices from the model parameters. The computations required are quite lengthy; therefore, a digital computer is required as the basic computation tool for the computational procedure. Since the use of the indices involves calculation of the indices for many network configurations, the procedure is required to be efficient with computer time. Without this efficiency, the indices would only be an interesting idea rather than a useful tool because the cost involved in using the indices would be prohibitive.

The indices are defined to be an indication of line loading and bus voltage problems. Therefore, the computational procedure includes a set of load flow equations. The non-linear equations describe the line flows and bus voltages when provided with line impedance, load and generation information. The basic equations are discrete in nature and describe the bus voltages and line flows for a specific set of load and generation data. Since the load and, therefore the generation, is not known exactly for the calculations described in this thesis, the basic equations are modified to calculate statistics of line flows and bus voltages when provided with statistics of the load. The line impedances are assumed known since these values do not vary significantly with time. Once the line flow and bus voltage statistics have been calculated, the probability of violating any of the network operational limits can be calculated. If lines are found which have a finite probability of overloading, an assessment of bus reliability is made. That is, not all buses will experience a service interruption because of any particular line being overloaded. This assessment is made by determining the buses served by the overloaded line, assuming these buses will experience service interruption because of the overload and assessing these buses with a probability of service interruption equal to the probability of overload. The buses being served by any given line can be determined with the use of sensitivity factors. These sensitivity factors were developed as a part of this thesis.

The reliability indices are computed by a three step process. First, a stochastic load flow calculation is performed for a particular load and network state. Second, the results of this calculation are then analyzed using sensitivity factors. Third, the results of step two are summed over all load and network states. Each of these steps is discussed in detail in this chapter.

Stochastic A.C. Load Flow

Load flow equations which describe the line flows and bus voltage within a network have existed for many years. These equations are nonlinear and solution of these equations requires an iterative procedure.

The Newton-Raphson method generally works quite well for this solution. These equations calculate a discrete set of bus voltages and line flows for a specific set of loads and generation.

The load model developed for this procedure does not provide a discrete load pattern. That is, the load is described as a function of the random variable ρ. (See Equation 8.) The statistics of ρ are known as well as the functional relationship which relates ρ to the load. To calculate the effects of a load described in this manner, a stochastic load flow is required. The stochastic load flow calculations are performed by linearizing the discrete load flow equations and calculating the statistics of the bus voltages and line flows using these linearized equations, the load equations (Equation 8), and the statistics of the random variable ρ.

For small networks, a single point of linearization is sufficient. For these networks this point is generally chosen to be the projected peak load for the period of the study.

For large systems, a single point of linearization generally does not supply sufficient accuracy for this procedure. The process to select these points of linearization is described below. The initial point of linearization is specified by the user. The subsequent points are chosen by increasing the load an amount equal to the average single unit generation ($\Delta\rho_s$). This point is tested to determine if it is greater than the upper limit of linearization from the previous point ($\rho > \rho_{max_{-1}}$). If this is false, the load (ρ) is increased again by the same amount. This process is repeated until a value of ρ is found. The generation dispatch and limits are calculated for this point. If the lower limit (ρ_{min}) is greater than the previous upper limit ($\rho_{max_{-1}}$), the value of ρ is reduced by an amount equal to half the previous increment. This process is repeated until the entire range of possible load values is covered.

Once a point of linearization is chosen, the stochastic load flow is a four step process: (1) the statistics of the generation are calculated as a function of the load; (2) a discrete load flow calculation is performed for the point of linearization; (3) the load flow equations are linearized about this point; (4) the probabilities of line overload and bus under-voltage are calculated directly from the statistics of the load and the linearized equations without actually calculating the statistics of the line flows and bus voltages. A flow chart depicting the stochastic load flow process is shown in Figure 3.

Generation Dispatch

The generation is a function of the load. The load is stochastic and therefore the generation is stochastic also. The generation is modeled as being dispatched according to an economic dispatch algorithm using a quadratic cost characteristic. This results in a generation pattern which is a linear function of the load subject to the generator limits. At the point where a generator reaches a limit, an abrupt change in generation dispatch occurs and a new set of linear equations must be calculated.

The total cost of generation is

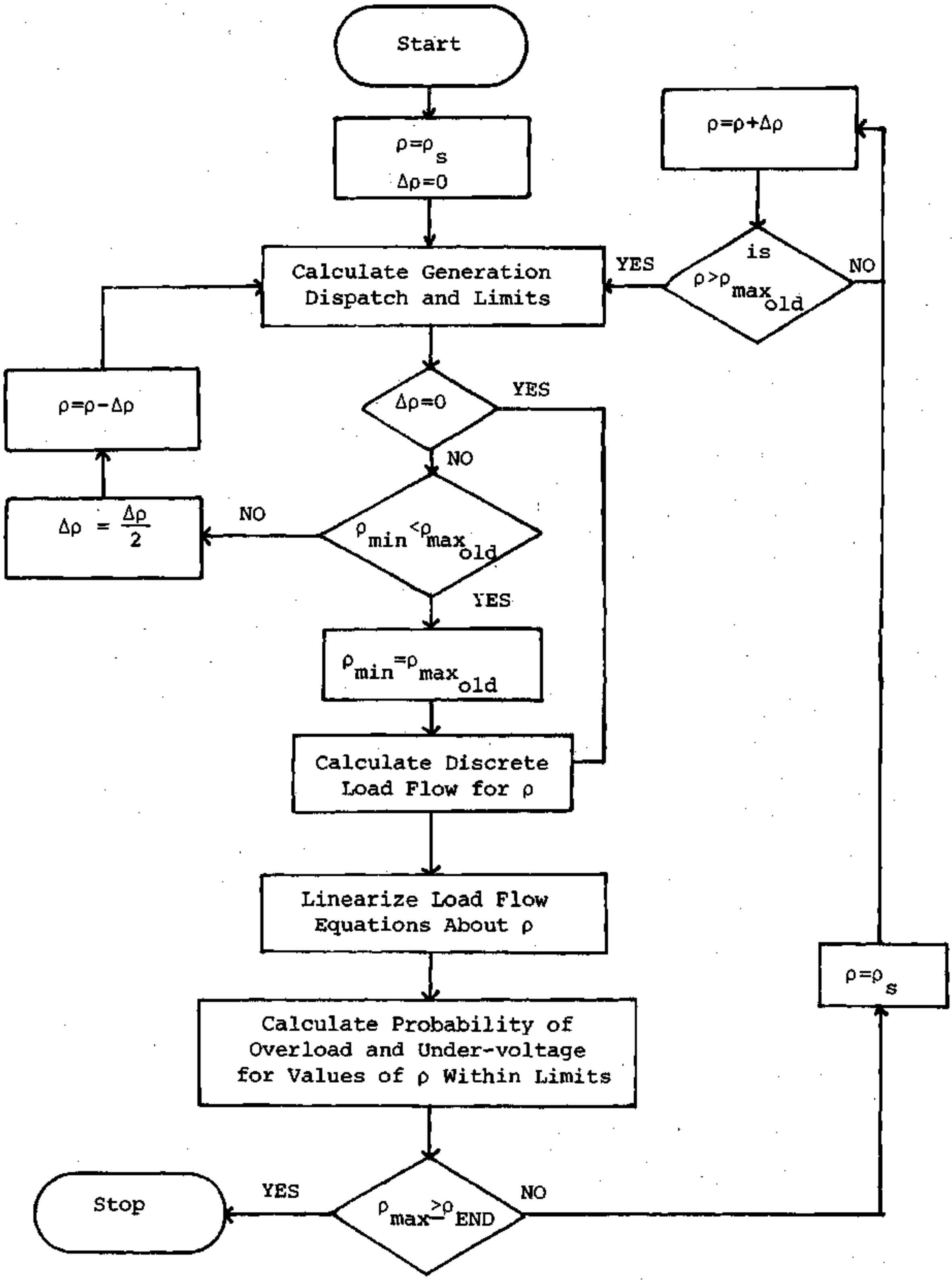

Figure 3. Flow Chart of Stochastic Load Flow Calculations

$$C_T = \sum_k C_k$$

$$= \sum_k a_{0k}+a_{1k}P_{Gk}+a_{2k}P_{Gk}^2 \tag{9}$$

where:

C_k	is the input cost in \$/hour for generator k
a_{0k}, a_{1k}, a_{2k}	are the cost coefficients for generator k
P_{Gk}	is the real power output of generator k.

The economic dispatch algorithm minimizes the total cost (C_T) subject to the following constraints:

$$\sum_k P_{Gk} = P_T = P_L + P_s \tag{10}$$

$$P_{Gk\ell} \le P_{Gk} \le P_{Gku}, \text{ for all } k \tag{11}$$

where:

P_T	is the total required generation
P_L	is the total load on the system
P_s	is the spinning reserve
$P_{Gk\ell}, P_{Gku}$	are the upper/lower bounds of operation for generator k.

Equations 9-11 imply:

$$\frac{\partial C_T}{\partial P_{G1}} = \frac{\partial C_T}{\partial P_{G2}} = \ . \ . \ . = \frac{\partial C_T}{\partial P_{Gk}} \tag{12}$$

These partial derivatives can be calculated from equation 9

$$\frac{\partial C_T}{\partial P_{Gk}} = a_{1k} + 2a_{2k}P_{Gk} \tag{13}$$

Combining equations 12 and 13 produce the following equations:

$$a_{11} + 2a_{21}P_{G1} = a_{1k} + 2a_{2k}P_{Gk} \tag{14}$$

for all k > 1. Equation 10 combined with the set of Equations 14 produce k linear equations with k unknowns which have the solution:

$$P_{Gk} = \frac{a_{11}-a_{1k}+2a_{21}P_{G1}}{2a_{2k}}, \quad k > 1 \tag{15}$$

$$P_{G1} = \frac{P_T-A_1}{A_2} \tag{16}$$

where:

$$A_1 = \sum_{k\neq 1} \frac{a_{11}-a_{1k}}{2a_{2k}}$$

$$A_2 = 1 + \sum_{k\neq 1} \frac{a_{21}}{a_{2k}}$$

Equations 15 and 16 are valid if the limits defined by equation 11 are not violated. The minimum and maximum values of P_T which define this valid region can be calculated from equations 11, 15, and 16.

$$P_{T_{min}} = \mathrm{Max}[P_{T_{min1}}, P_{T_{min2}}, \ldots, P_{T_{mink}}]$$

$$P_{T_{max}} = \mathrm{Min}[P_{T_{max1}}, P_{T_{max2}}, \ldots, P_{T_{maxk}}] \tag{17}$$

where:

$$P_{T_{min1}} = A_1 + A_2 P_{G1\ell}$$

$$P_{T_{mink}} = \frac{A_2}{2a_{21}} (2a_{2k} P_{Gk\ell} - a_{11} + a_{1k}) + A_1$$

$$P_{T_{max1}} = A_1 + A_2 P_{G1u}$$

$$P_{T_{maxk}} = \frac{A_2}{2a_{21}} (2a_{2k} P_{Gk\ell\ell} - a_{11} + a_{1k}) + A_1.$$

For values of P_T outside the limits defined by equation 17, the limiting generator must be set to the limiting value and the dispatch process repeated.

Discrete A.C. Load Flow

The second step in the stochastic load flow calculation is a discrete load flow calculation to determine a point of linearization. The input for the stochastic load flow is the load vector of the system load model.

$$\overline{S} = \overline{P} + j\overline{Q} = (\overline{\alpha} + j\overline{\beta})\rho \tag{18}$$

where ρ is a gaussian random variable with mean μ and variance σ^2. The input for the discrete load flow is calculated by setting $\rho=\rho_s$. This produces a discrete load vector which is the mean of the stochastic load vector.

The computational procedures developed for this thesis are constrained to be efficient with computer time. Since the discrete load flow is a time-consuming process which is repeated many times in the calculation of the indices, an efficient discrete load flow algorithm is required. The load flow procedure presented by Stott and Alsac[31,32] is used to perform the discrete load flow calculations. This procedure is very fast for moderately accurate load flows.

This discrete load flow procedure decouples the P-θ equations from the Q-V equations. This decoupling produces a significant decrease in the computer time required for each iteration. The accuracy of each iteration is reduced which results in more iterations being required. However, it has been experimentally determined that a reduction in the total time required for a solution is achieved. An additional advantage of this method is a reduced core storage requirement since more zero terms occur in the matrix for the decoupled equations.

The development of the decoupled load flow begins with the standard polar power-mismatch equations:

$$\begin{bmatrix} \overline{\Delta P} \\ \overline{\Delta Q} \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \overline{\Delta \theta} \\ \overline{\Delta V}/V \end{bmatrix} \tag{19}$$

These equations represent a linearization of the load flow equations about an assumed operating point. The decoupling principle neglects the contribution of the off-diagonal submatrices N and J. This assumption produces the following equations.

$$\overline{\Delta P} = H\ \overline{\Delta \theta} \tag{20}$$

$$\overline{\Delta Q} = L\ \overline{\Delta V}/V \tag{21}$$

Further assumptions can be made regarding the elements of H and L. These are:

$$\cos \theta_{km} \simeq 1$$

$$G_{km} \sin \theta_{km} << B_{km}$$

$$Q_k << B_{kk} V_k^2$$

where:

$G_{km} + jB_{km}$	is the (k,m) the element of the bus admittance matrix
$\theta_{km} = \theta_k - \theta_m$	is the angular difference between the voltages at buses k and m
$P_k + jQ_k$	is the scheduled complex power at bus k.

Equations 20 and 21 can now be written:

$$\overline{\Delta P} = [V\ B'\ V]\ \overline{\Delta\theta} \qquad (22)$$

$$\overline{\Delta Q} = [V\ B''\ V]\ \overline{\Delta V}/V$$

The elements of B' and B" are strictly elements of [-B]. However, it has been found that convergence is improved by:

(1) Omitting from B' the effects of network elements that predominately affect reactive power flows, i.e. shunt reactances and off-nominal, in-phase transformer taps.

(2) Omitting from B" the angle shifting effects of phase shifters.

If the left V terms are shifted to the left side of the equations while making the approximation V=1 for the right V in equation 22, the following expressions result.

$$\overline{\Delta P}/V = B'\ \overline{\Delta\theta} \qquad (24)$$

$$\overline{\Delta Q}/V = B''\ \overline{\Delta V} \qquad (25)$$

The matrices B' and B" are real, sparse, and constant. Sparsity techniques can be used and triangulation of the matrices is required only once.

The algorithm involves a 1θ-$1V$ iteration scheme. That is, the bus voltage angles and magnitudes are calculated alternatively using the most recent phase and magnitude values. The process is continued until the following conditions are satisfied.

$$\max|\Delta P| \leq C_p \qquad \max|\Delta Q| \leq C_q \tag{26}$$

where:

C_p is the maximum permissible real power mismatch

C_q is the maximum permissible reactive power mismatch

If a generator cannot maintain its prescribed voltage without violating its VAR limits, the associated bus is converted to a P-Q type bus with Q being set to the limiting value. This requires that B" matrix be retriangulated.

To study the effects of a branch outage, the B matrices must be altered as follows:

$$B_1 = B_0\, \overline{M}\, \overrightarrow{M}^{T} \tag{27}$$

where:

B_1 represents the new B matrix

B_0 represents the original B matrix

b is the nominal series susceptance

$\overline{M}$ is a column vector which is null except for $M_k = a$ and $M_m = -1$

a is the off nominal turns ratio referred to the bus corresponding to row m, for a transformer

= 1, for a line

By the matrix inversion lemma

$$B_1^{-1} = B_0^{-1} - c\overline{X}\,\overline{M}^{\tau} \qquad (28)$$

where:

$$\overline{X} = B_0^{-1}\,\overline{M}$$

$$c = (1/b + \overline{M}^{\tau}\overline{M})^{-1}$$

If $1/c = 0$, a split network is indicated.

<u>Partially Coupled Load Flow</u>

The discrete load flow provides the point of linearization for the stochastic load flow procedure. The next step in the stochastic load flow procedure is to linearize the load flow equations about this point. Equation 19 describes the direct linearization of these equations. It was shown in the previous section that certain simplifying assumptions can increase the efficiency of these equations. The decoupled equations do not provide sufficient accuracy for the linearization procedure. Therefore, a procedure is required to approach the efficiency of the decoupled equations without losing the accuracy of the original linearized equations. These two requirements are met by developing a procedure which uses the N=0 assumption but does not alter J. The further assumptions of the decoupled method are also used. The following equation is a result of these assumptions.

$$\begin{bmatrix} \frac{\overline{\Delta P}}{V} \\ \\ \frac{\overline{\Delta Q}}{V} \end{bmatrix} = \begin{bmatrix} B' & 0 \\ \\ J & B'' \end{bmatrix} \begin{bmatrix} \overline{\Delta\theta} \\ \\ \overline{\Delta V} \end{bmatrix} \tag{29}$$

This equation can be solved with the use of the matrix inversion lemma:

$$\begin{bmatrix} \overline{\Delta\theta} \\ \\ \overline{\Delta V} \end{bmatrix} = \begin{bmatrix} B'^{-1} & 0 \\ \\ -B''^{-1}JB^{-1} & B''^{-1} \end{bmatrix} \begin{bmatrix} \overline{\Delta P}/V \\ \\ \overline{\Delta Q}/V \end{bmatrix} \tag{30}$$

where the elements of J are calculated

$$j_{km} = (G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km})V_m \tag{31}$$

The procedure then becomes:

$$\overline{\Delta\theta} = B'^{-1}[\frac{\overline{\alpha} - \overline{G}}{V}]\Delta\rho = \overline{X}\,\Delta\rho \tag{32}$$

$$\overline{\Delta V} = -B''[J\overline{X} - \overline{B}/V]\Delta\rho = \overline{Z}\,\Delta\rho \tag{33}$$

The vector $\overline{X}$ is calculated by a simple matrix-vector multiplication and the vector $\overline{Z}$ by a vector addition and a matrix-vector multiplication. These equations are almost as fast as the decoupled equations and provide much greater accuracy.

The vector $\overline{G}$ in Equation 32 represents the effects of generator control. That is, as the load is increased, the generation is increased accordingly. $\overline{G}$ is calculated from the generation dispatch coefficients.

$$\overline{G} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial P_{G1}}{\partial \rho} \\ \dfrac{\partial P_{G2}}{\partial \rho} \\ \dfrac{\partial P_{Gk}}{\partial \rho} \end{bmatrix} \tag{34}$$

The zeros correspond to buses without generation. The term $\dfrac{\partial P_{Gk}}{\partial \rho}$ appears in the location corresponding to the bus where generator k is located, and is calculated as follows:

$$\frac{\partial P_{Gk}}{\partial \rho} = \frac{\partial P_{Gk}}{\partial P_T} \frac{\partial P_T}{\partial \rho} \tag{35}$$

where:

$$\frac{\partial P_T}{\partial \rho} = |\bar{\alpha}| = \sum_i \alpha_i$$

and

$$\frac{\partial P_{G1}}{\partial P_T} = \frac{1}{A_2} \tag{36}$$

$$\frac{\partial P_{Gk}}{\partial P_T} = \frac{\partial P_{Gk}}{\partial P_{G1}} \frac{\partial P_{G1}}{\partial P_T} \tag{37}$$

$$\frac{\partial P_{Gk}}{\partial P_{G1}} = \frac{a_{21}}{a_{2k}} \tag{38}$$

Calculation of Under-Voltage Probabilities

For each bus k, the probability of under-voltage can now be calculated for the particular load and network states being considered.

$$P_r[V_k < V_{kc}] = P_r[\rho > \rho_{ck}] \tag{39}$$

$$\rho_{ck} = \mu + \frac{V_{kc} - V_{km}}{Z_k} \tag{40}$$

where:

V_k is voltage magnitude at bus k

V_{km} is voltage magnitude at bus k for $\rho=\mu$

V_{kc} is value of voltage at bus k below which is considered an unsatisfactory condition

z_k is k^{th} element of $\bar{Z}$ (see equation 33).

Calculation of Overload Probabilities

To calculate line flows the expression for real and reactive power flows are linearized but the quadratic nature of total line flow is retained. The expressions for line flows from bus x to bus y are given below.

$$P_{xy} = G_{xy}V_x^2 - V_xV_y(G_{xy}\cos\theta_{xy} + B_{xy}\sin\theta_{xy})$$

$$Q_{xy} = -B_{xy}V_x^2 - V_xV_y(G_{xy}\sin\theta_{xy} - B_{xy}\cos\theta_{xy}) \qquad (41)$$

where:

$G_{xy} + jB_{xy}$ is the line admittance

V_x, V_y are the bus voltage magnitudes

and

$\theta_{xy} = \theta_x - \theta_y$ is the voltage angular difference.

These equations can be linearized as follows:

$$\Delta P_{xy} = 2G_{xy}V_xV_x + V_xV_y(G_{xy}\ \sin\theta_{xy} - B_{xy}\ \cos\theta_{xy})(\Delta\theta_x - \Delta\theta_y)$$

$$- (V_y\Delta V_x + V_x\Delta V_y)(G_{xy}\ \sin\theta_{xy} + B_{xy}\ \sin\theta_{xy})$$

$$\Delta Q_{xy} = -2B_{xy}V_x\Delta V_x - V_xV_y(G_{xy}\ \cos\theta_{xy} + B_{xy}\ \sin\theta_{xy})(\Delta\theta_x - \Delta\theta_y)$$

$$- (V_y\Delta V_x\Delta V_y)(G_{xy}\ \sin\theta_{xy} - B_{xy}\ \cos\theta_{xy}) \qquad (42)$$

If this line contains a transformer set at an off-nominal tap, ΔQ_{xy} must be corrected as follows:

$$\Delta Q_{xy} = \Delta Q_{xy}/a - 2V_x(1/a)(1/a - 1)B_{xy}\Delta V_x,$$

if x is the primary bus

$$\Delta Q_{xy} = \Delta Q_{xy}/a - 2V_x(1-1/a)B_{xy}\Delta V_x, \qquad (43)$$

if x is the secondary bus, and where a is the off-nominal turns ratio referred to the primary bus.

To calculate $\rho_{\ell c}$, the value of ρ at which the line ℓ overloads, the following expression must be evaluated.

$$\rho_{\ell c} = \mu - \frac{(P\Delta P + Q\Delta Q) + \sqrt{(P\Delta P + Q\Delta Q)^2 - (\Delta P^2 + \Delta Q^2)(P^2 + Q^2 - k_\ell^2)}}{\Delta P^2 + \Delta Q^2} \qquad (44)$$

where:

k_ℓ is the rated capacity of line ℓ.

The probability of line ℓ overloading is:

$$P_r[S_\ell > K_\ell] = P_r[\rho > \rho_{\ell c}] \tag{45}$$

Equation 45 is valid for values of ρ which represent system loads within the limits defined by the generation dispatch algorithm. For values of ρ outside this region, another point of linearization must be chosen and the stochastic load flow calculations performed again.

Sensitivity Factors

The function of a power transmission network is to carry electric power from the generators to the load. Therefore, the points of final interest are the load buses. That is, the internal functions are only of significance for their effect on load carrying ability of the network. The computational procedures presented in the previous section can calculate the probability of an overloaded line or a bus that is not within specified voltage limits. However, the question of the effect of the overloaded line on the total operation of the network must be answered.

A power transmission network is a highly complex network. Generally there are many paths for power to flow from the generators to the load and the loss of one path may not affect the ability of the network to satisfy the load. An additional factor adding to the complexity of the power system is the network controller. The network controller monitors line flows and when he detects an overloaded line he will perform a control action designed to alleviate the overload. The particular action taken by the controller is a function of many unpredictable factors including the human factor. However, whatever the control action is, it usually results in a loss of load at one or more of the buses being supplied by that line. Therefore, although the control action itself is highly complex, the effects of this action can be predicted in an average manner if the buses supplied by the line can be identified.

In order to solve this problem, a line-flow sensitivity factor is introduced. The incremental line flow due to incremental load at a bus is calculated. The lines which have incremental line flow in the direction of current flow are carrying power to the bus. Conversely, if incremental power flow at a bus tends to decrease current flow in a line, that line is carrying power from that bus.

Generators are the source of the power used at the load points. However, using the standard polar-mismatch load flow equations, generators are not a source of incremental real power. Generators are a source of base load real power and incremental reactive power. The slack bus is the only source of incremental real power. Since the problem here deals with base case real power rather than incremental power, the reactive power flow equations must be used.

The sensitivity factor of line flow in line r to power at bus k is defined as

$$S_{rk} = \frac{\partial Q_r}{\partial Q_k} \tag{46}$$

where

Q_r is reactive power in line r

Q_k is reactive power flow at bus k.

The current in line r can be calculated as

$$Q_r = \mathrm{Im}\{E_{r1} I_r^*\} \tag{47}$$

$$= \mathrm{Im}\{E_{r1}(E_{r1}-E_{r2})^* Y_r^*\} \tag{48}$$

where:

E_{r1}, E_{r2} are the complex bus voltages at the ends of line r

Y_r is the complex admittance of line r $= G_r + jB_r$

Im is the imaginary part of a complex number

* denotes conjugation.

Equation 48 can be expanded as follows:

$$Q_r = \mathrm{Im}\{|E_{r1}|(|E_{r1}|-|E_{r2}|\cos\theta_{12} + j|E_{r2}|\sin\theta_{12})(G-jB)\} \tag{49}$$

where:

$$\theta_{12} = \theta_{r1} - \theta_{r2}$$

$$= \mathrm{Arg}(E_{r1}) - \mathrm{Arg}(E_{r2}).$$

Making the assumptions:

$$G \ll B$$

$$\sin\theta_{12} \simeq 0$$

$$\cos\theta_{12} \simeq 1$$

Equation 49 can be simplified to:

$$Q_r = -B(|E_{r1}|)(|E_{r1}|-|E_{r2}|). \tag{50}$$

Differentiating equation 50:

$$S_{rk} = \frac{\partial Q_r}{\partial Q_k} = -B([2|E_{r1}|-|E_{r2}|]\ \frac{\partial|E_{r1}|}{\partial Q_k} - |E_{r1}|\ \frac{\partial|E_{r2}|}{\partial Q_k}). \tag{51}$$

Making the assumptions

$$|E_{r1}| \simeq |E_{r2}| \simeq 1.$$

$$S_{rk} = \frac{\partial Q_r}{\partial Q_k} = -B(\frac{\partial|E_{r1}|}{\partial Q_k} - \frac{\partial|E_{r2}|}{\partial Q_k}). \tag{52}$$

These partial derivatives can be approximated using the elements of the B''^{-1} matrix.

$$S_{rk} \simeq -B(b''^{-1}_{r1,k} - b''^{-1}_{r2,k}) \tag{53}$$

where:

$b''^{-1}_{rl,k}$ is the element of the B''^{-1} matrix in the rl row and k column

If the element $b''^{-1}_{rl,k}$ does not exist, then

$$\frac{\partial |E_{rl}|}{\partial Q_k} = 0.$$

Sensitivity factors of all lines are calculated for each bus. This calculation is performed once using the B" matrix with all generators on and within reactive power limits. The X most sensitive lines with sensitivity greater than 0.1 are chosen for each bus. This is equivalent to choosing only lines which carry more than 10% of the load to the bus being considered. The limit of X lines was chosen since only finite memory is avaialbe. The number X must be chosen as a function of the network being considered. For the 90 bus sample network used for this research, a value of eight is used for X. The reliability calculations remain relatively constant for values of X between six and ten. The insensitivity to this variable is to be expected since at some level away from the bus, the network will resemble an infinite bus in the reliability sense.

Calculation of Indices

The approximate values for the indices defined in Equations 3 and 4 can now be calculated as shown in Figure 4.

$$p_k = \frac{\text{expected value of total duration of load loss}}{\text{total time}}$$

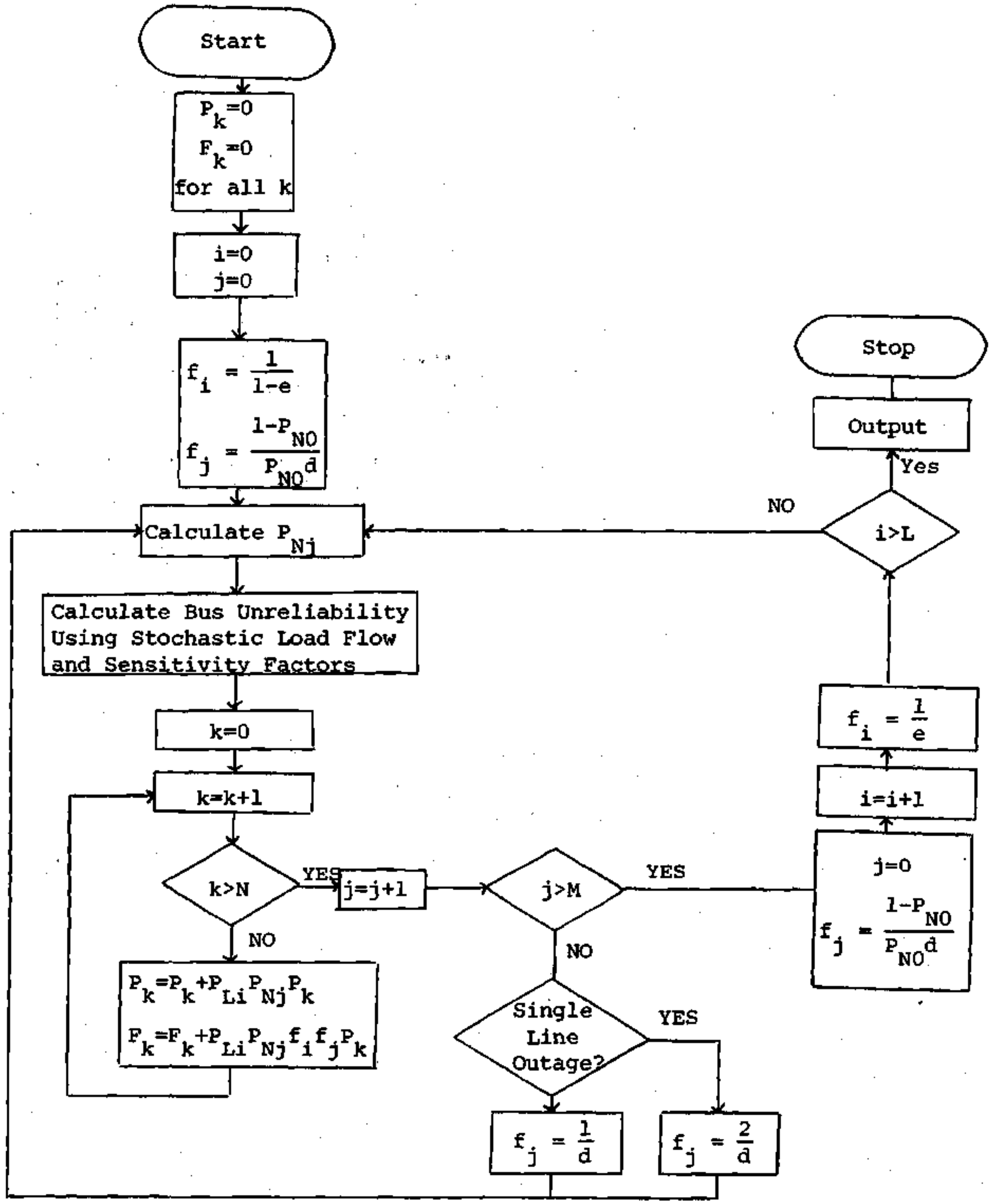

Figure 4. Flow Chart of Computational Procedure.

$$= \sum_{i=0}^{L} \sum_{j=0}^{M} P_{Li} P_{Nj} P_{ijk} \tag{54}$$

where:

P_{Li} is the probability that load state i exists

P_{Nj} is the probability that network state j exists

L is the number of load states

M is the number of network states

P_{ijk} is the conditional probability that the load at bus k will not be satisfied conditioned on load state i and network state k

$= \mathrm{Max}\{P_r[V_k < V_{kc}], P_r[S_{\ell_{k1}} > K_{\ell_{k1}}] \; . \; . \; . \; , P_r[S_{\ell_{kX}} > K_{\ell_{kX}}]\}$

$\ell_{k1} - \ell_{kX}$ are the lines with largest current sensitivity factors to power flow at bus k

f_k expected frequency of load loss

$$= \sum_{i=0}^{L} \sum_{j=0}^{M} F_{ij} P_{ijk} \tag{55}$$

where:

F_{ij} is the frequency of occurrence of the combined event load state i and network state j

$F_{ij} = P_{Li} P_{Nj} (1/e+1/d)$, $i \neq 0$, $j \neq 0$ for line outage

$F_{ij} = P_{Li} P_{Nj} (1/e+2/d)$, $i \neq 0$, $j \neq 0$ for bus outage

$F_{0j} = P_{L0} P_{Nj} (1/1-e+1/d)$, $j \neq 0$ for line outage

$F_{0j} = P_{L0} P_{Nj} (1/1-e+2/d)$, $j \neq 0$ for bus outage

$F_{i0} = P_{Li} P_{N0} (1/e + 1-P_{N0}/P_{N0} d)$, $i \neq 0$

$F_{00} = P_{L0} P_{N0} (1/1-e+1-P_{N0}/P_{N0} d)$

Summary of Computational Procedures

The goal of the research presented here is to develop a procedure to calculate a set of indices which will be an accurate indication of the reliability of a given power transmission network. The procedure is based on a set of decoupled load flow equations. These equations are the most efficient method available for a moderately accurate, deterministic load flow. The equations are then partially re-coupled to linearize the system about the calculated operating point. The partially coupled equations require more computer time but provide a higher degree of accuracy than can be obtained from the decoupled equations. This linearization is used to perform stochastic load flow calculations.

The results of the stochastic load flow equations are analyzed with the use of sensitivity factors and Markov theory to calculate two indices per load point. These indices indicate the reliability of the network at that point.

The procedure is implemented on a digital computer. The program interfaces with existing load flow data and load prediction data.

CHAPTER IV

PROPERTIES OF THE PROCEDURE

The concept of reliability was introduced in Chapter II. Chapter III presented a computational method to quantitatively measure the reliability of a power transmission network. If these indices are to accurately measure reliability, they must exhibit certain characteristics. These characteristics include sensitivity to network parameters and consistency with network operation. Another desirable feature is that errors in the component reliability data do not destroy the consistency of the indices. This additional feature is desirable because component reliability data is very difficult to measure. Therefore, errors exist in the available data which cannot be avoided. Sensitivity to network parameters is desirable because the use of the index is used as a tool in evaluting networks which vary only slightly. The consistency with network operation is required by the definition of reliability.

In addition to the above properties of the indices, computational efficiency and accuracy are required of the procedure. The procedure is also required to be applicable to a large network and use commonly existing data. Chapter V demonstrates the computational aspects of the procedure, whereas this chapter analyzes the consistency and sensitivity to data errors.

Consistency

The indices presented in this thesis are intended primarily for use in power system planning. There are two situations in which the indices will be useful: (1) bus reliability evaluation for the existing network; (2) comparative reliability evaluation for proposed networks. Bus evaluation for the existing network will be determined by first computing the reliability indices for all load buses in the network. These indices will then be compared with a minimum acceptable standard which will be derived from past operating experience. If a particular bus or group of buses do not meet the standard of the operating company, the planner will know that some form of network expansion is required.

Once the need for network expansion has been identified, the particular form of expansion must be determined. The network planner will use information obtained from the indices for the current network along with past experience and engineering judgement to develop a set of proposed network expansion plans. Only one of these plans can be used, therefore, the best plan must be identified and implemented. Comparative network reliability evaluation will be used in the evaluation of these expansion plans. This is done by first computing indices for the buses of interest for all proposed network configurations. These indices can then be compared to determine which of the proposed expansion plans is best in terms of reliability.

Both of the uses described above require that the indices be used in a relative manner. For the evaluation of the existing network, reliability indices will be compared against indices of other buses which are defined to be acceptable. The evaluation of network expansion plans requires that values of the indices be compared against the same indices calculated using other network configurations. The indices for the expansion plans will also be compared with the standard which is used in the evaluation of the existing network. Because of the manner in which the indices will be used, two requirements must be satisfied: (1) changes in the indices should be consistent with changes in the input; (2) the computational procedures used should not introduce inaccuracies which destroy the consistency of the indices.

The consistency of the indices can easily be shown. Equation 54 states that bus reliability is a function of three quantities: (1) load state probability; (2) network contingency probability; (3) probability of overload or under-voltage. If the duration of the peak load state increases, the bus reliability decreases. If the duration of the network contingency increases, the bus reliability decreases. If the load decreases or an additional feeder line is added, the probability of overload decreases and, therefore, the bus reliability increases. All of these relationships are consistent with network operation. Proof of this consistency is shown in Appendix I. A similar argument can be made for the frequency index.

The models have been formulated in such a way that the computational procedure is an exact calculation with the exception of the linearization required for the stochastic load flow. Although the load flow equations are non-linear, the equations are monotonic and continuous in a small region about the point of linearization. Therefore, the linearization does not alter the consistency of the model and the computational procedure is consistent with network operation.

Sensitivity of the Indices

The computational procedures presented here have assumed that failure statistics for network components are known. However, since elements of transmission systems are very reliabile, failure statistics for these elements are difficult to measure. The following analysis recognizes this difficulty and calculates statistics of the indices when the failure statistics are treated as random variables.

Line failure statistics are commonly measured by assuming that the failure rate is directly proportional to the line length and that all lines of similar construction have the same failure rate per mile. The bus failure rate is more difficult to measure. However, the failure rate for buses of similar construction will be either a constant or a function of the number of lines connected to the bus. Therefore, for a single voltage network with uniform line construction, Equations 54 and 55 can be reduced to:

$$P_k = \alpha_{Pk} P_\ell + \beta_{Pk} P_\beta \tag{56}$$

$$f_k = \alpha_{Fk} F_\ell + \beta_{Fk} F_\beta \tag{57}$$

where:

P_ℓ is the line failure probability (outage duration/year-mile)

P_β is the bus failure probability (outage duration/year or outage duration/year-line)

F_ℓ is the line failure frequency (outages/year-mile)

F_β is the bus failure frequency (outages/year or outages/year-line).

The quantities α_{Pk}, β_{Pk}, α_{Fk}, β_{Fk} are values calculated by the computational procedure. Treating the failure rates as independent random variables, the density function for P_k can be written as:[33]

$$f_{Pk}(P_k) = \frac{1}{|\alpha_{Pk}\beta_{Pk}|} \int_{-\infty}^{\infty} F_{P\ell}\left(\frac{P_{k-x}}{\alpha P_k}\right) f_{P\beta}\left(\frac{x}{\beta P_k}\right) dx \tag{58}$$

where:

f_{Pk} is the density function for P_k

$f_{P\ell}$ is the density function for P_ℓ

$f_{P\beta}$ is the density function for P_β.

If P_ℓ and P_β are assumed normal, then:

$$\mu_{Pk} = \alpha_{Pk}\mu_{P\ell} + \beta_{Pk}\mu_{P\beta}$$

$$\sigma^2_{Pk} = \alpha^2_{Pk}\sigma^2_{P\ell} + \beta^2_{Pk}\sigma^2_{P\beta}$$

where:

μ_{Pk} is the mean value of P_k

$\mu_{P\ell}$ is the mean value of P_ℓ

$\mu_{P\beta}$ is the mean value of P_β

σ^2_{Pk} is the variance of P_k

$\sigma^2_{P\ell}$ is the variance of P_ℓ

$\sigma^2_{P\beta}$ is the variance of P_β

Since power transmission networks are highly reliable, the values of α_{Pk} and β_{Pk} will be small and therefore σ^2_{Pk} will be smaller than $\sigma^2_{P\ell}$ and $\sigma^2_{P\beta}$.

Since the usage of the indices will be as a relative set of numbers, joint statistics are also of interest. The joint density function for P_x, P_y (failure indices for buses x and y) can be written as:[33]

$$f_{Px,Py}(P_x,P_y) = \frac{1}{|\alpha_{Px}\beta_{Py}-\alpha_{Py}\beta_{Px}|} f_{P\ell,P\beta}(a_1P_x+b_1P_y, c_1P_x+d_1P_y) \qquad (59)$$

where:

$$P_x = \alpha_{Px}P_\ell + \beta_{Px}P_\beta$$

$$P_y = \alpha_{Py}P_\ell + \beta P_y P_\beta$$

$$P_\ell = a_1P_x + b_1P_y$$

$$P_\beta = c_1P_x + b_1P_y$$

$f_{Px,Py}$ is the joint density function of P_x, P_y

$f_{P\ell,P\beta}$ is the joint density function of P_ℓ, P_β.

If P_ℓ and P_β are assumed to be independent,

$$f_{Px,Py}(P_x,P_y) = \frac{1}{|\alpha_{Px}\beta_{Py}-\alpha_{Py}\beta_{Px}|} f_{P\ell}(a_1P_x+b_1P_y)f_{P}(c_1P_x+d_1P_y) \qquad (60)$$

If P_ℓ and P_β are also assumed to be jointly normal, the following equations result:

$$\mu_{Px} = \alpha_{Px}\mu_{P\ell} + \beta_{Px}\mu_{P\beta}$$

$$\mu_{Py} = \alpha_{Py}\mu_{P\ell} + \beta_{Py}\mu_{P\beta}$$

$$\sigma^2_{Px} = \alpha^2_{Px}\sigma^2_{P\ell} + \beta^2_{Px}\sigma^2_{P\beta}$$

$$\sigma^2_{Py} = \alpha^2_{Py}\sigma^2_{P\ell} + \beta^2_{Py}\sigma^2_{P\beta}$$

$$r_{Px,Py}\sigma_{Px}\sigma_{Py} = \alpha_{Px}\alpha_{Py}\sigma^2_{P\ell} + \beta_{Px}\beta_{Py}\sigma^2_{P\beta}$$

A degree of confidence in the comparison of buses x and y can then be calculated by the following equation:

$$Pr[P_x > P_y] = \int_{-\infty}^{\infty}\int_{Px}^{\infty} f_{Px,Py}(P_x,P_y)d_{Py}d_{Px}$$

$$= \frac{1}{2\pi\sigma_{Px}\sigma_{Py}\sqrt{1-r^2_{Px,Py}}}\int_{-\infty}^{\infty}\int_{P_x}^{\infty}$$

$$\exp\left[\frac{(P_x-\mu_{Px})^2}{\sigma^2_{Px}} - \frac{2r_{Px,Py}(P_x-\mu_{Px})(P_y-\mu_{Py})}{\sigma^2_{Py}}\right.$$

$$\left. + \frac{(P_y - \;_{Py})^2}{\sigma^2_{Py}}\right]dp_y dp_x. \qquad (61)$$

The values for α_{Px}, β_{Px}, α_{Py}, β_{Py} are generated by the procedure. The values for $\mu_{P\ell}$, $\mu_{P\beta}$, $\sigma^2_{P\ell}$, $\sigma^2_{P\beta}$ must be supplied as input.

CHAPTER V

EVALUATION

Introduction

This thesis develops a set of indices which are a quantitative indication of the reliability of a power transmission network. The computational procedure to calculate these indices has also been developed. However, before those indices can be considered to be useful, an evaluation is necessary to demonstrate the usefulness of the method and its superiority to previously existing methods.

This evaluation has been performed. The evaluation considered three test cases. The first two tests used networks which have appeared previously in the literature of network reliability. The method of this thesis was compared to the methods presented in the articles associated with the test case. The third test case used a portion of the Georgia Power Network. This test case was chosen in order to demonstrate the applicability of the method to a large realistic network. Since no other existing direct computational method can be applied to a large network, the evaluation of the results of this case was twofold: (1) demonstration that the method will produce a set of indices at reasonable computation cost; (2) comparison of the indices with a set of approximate indices calculated using a Monte Carlo simulation of the operation of the network.

Comparison with Three Bus Network

The initial evaluation of this method is performed by comparing the indices calculated by this method with those of other methods. Bhavaraju and Billinton[23] perform a comparison of three methods on a three bus network. This network is shown in Figure 5. Table 2 gives the failure rates associated with these lines. The load data for this network is given in Table 3. The method presented here (Conditional Probability with load) is compared with the other methods in Table 4. This table is an extension of Table VI of reference 23. Since none of the previous methods required line impedance data, none was given in reference 23. Therefore, line impedances are calculated using two different assumptions. The methods are compared using both 80 and 100 MW line capacity.

The most favorable comparison occurs if all lines are assumed to be of equal impedance. This causes the line flows to distribute evenly which is the most desirable flow distribution for all lines of equal capacity as they are here. This is the best basis for comparison of the methods since the previous methods assume the most favorable line flow possible.

Another comparison which is made involves using different line impedances. The assumption is made that line failures are proportional to line length. Line impedances are also assumed proportional to line length. Using these two assumptions, a more realistic set of line impedances is calculated. Using these impedances, the results are quite different.

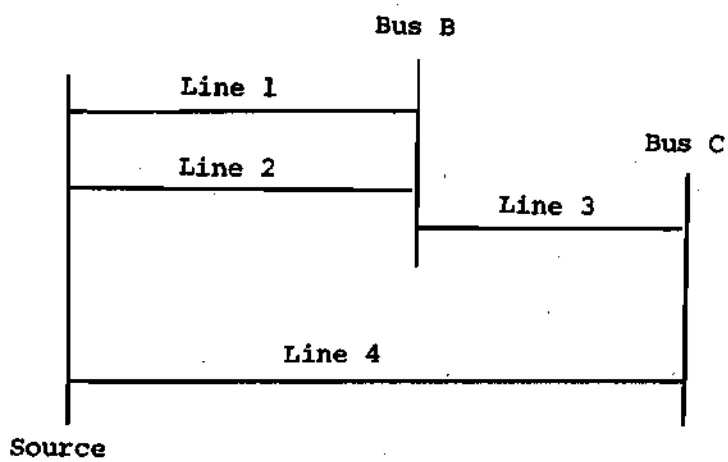

Figure 5. Three bus network.

Table 2. Basic Failure Rates for Three Bus Network

Line	Failures/Year	Failures/Day	Expected Repair Duration (Hours)
1 or 2	0.5	.001370	7.5
3	0.1	.000274	7.5
4	0.6	.001644	7.5

Table 3. Load Data for Bus B and Bus C of Three Bus Network

Number of days in the period	D = 20
Expected duration of peak load	e = 0.5 day
Low load level L_o	= 50 MW
Probability of low load existing in the period	= 1-e=0.5

Peak Load Level L, MW	Number of Days L Occurs, n_l	Probability of L Existing in the Period A_L	Variance of Load Level σ^2
100	4	0.1	0
90	4	0.1	0
80	4	0.1	0
70	4	0.1	0
60	4	0.1	0

Table 4. Reliability Indices for Three Bus Network

Method	Loss of Load Measure	100 MW Capacity Bus B	100 MW Capacity Bus C	80 MW Capacity Bus B	80 MW Capacity Bus C
Modified Conditional Prob. Method (Equations 4 and 5)	Frequency	.000955	.001060	.520	.552
	Duration	2.9	2.9	4.6	4.6
Extended Method 2	Frequency	.000955	.001060	.520	.533
	Duration	2.9	2.9	4.6	4.6
Conditional Prob. Method (Equation 2)	Frequency	.000728	.000831	.320	.341
All Methods (Equations 1 and 3)	Prob.	$.312\times10^{-6}$	$.357\times10^{-6}$	$.274\times10^{-3}$	$.291\times10^{-3}$
	Expectation	.000227	.000243	.200	.212
Conditional Prob. with Load (equal line impedances)	Frequency	.000784	.000135	.552	.552
	Prob.	$.256\times10^{-6}$	$.44\times10^{-7}$	$.291\times10^{-3}$	$.291\times10^{-3}$
	Duration	2.9	2.9	4.6	4.6
Conditional Prob. with Load	Frequency	.162	.162	.715	.715
	Prob.	$.86\times10^{-4}$	$.86\times10^{-4}$	$.377\times10^{-3}$	$.377\times10^{-3}$
	Duration	4.6	4.6	4.6	4.6

Duration is in hours.
Frequency is in occurrences per year
Expectation is in days per year

Two conclusions are drawn from this comparison. First, reliability calculations are not independent of line impedances. This is demonstrated by the fact that the indices changed significantly when realistic line impedances are used rather than equal line impedances. Second, methods which do not account for flow distribution only calculate an upper bound on the actual reliability of the network. As demonstrated, this bound can be significantly greater than the actual reliability of the network. The method presented in this thesis considers the effect of line impedance on both line flows and bus voltages. This method is, therefore, more accurate than any of the methods listed in Table 4 and therefore more desirable.

Comparison with Five Bus Network

One method which has appeared in the literature which uses line impedances is presented by Billinton and Bhavaraju.[21] In this reference the authors propose a five bus network to use for sample calculations. The method presented in this thesis is compared with the method of reference 21 using this five bus network (Figure 6). The generation load, and line data are given in Tables 5 and 6.

For this network the line capacities used for this network exceed the total load on the network. Therefore, disconnected buses and bus voltages exceeding limits are the only factors contributing to bus unreliability. In this network the generation is approximately 100 miles away from the points of load. Therefore, bus voltages are heavily dependent on VAR scheduling. A VAR schedule is not given in reference 21, therefore, one was assumed that approximated the results presented in reference 21. For this study the VAR scheduling was established to control the voltage of Bus 1 to 1.01 pu and Bus 2 to 1.04 pu. If generation VAR limits were exceeded, the generators were set to the limit and the corresponding bus voltage allowed to fluctuate. The load duration at each bus was represented by a normalized load duration curve, approximated by a single straight line from the 100- to the 40-percent load points. That is, a uniform load distribution was used rather than the gaussian model proposed in Chapter II.

Table 7 presents the results of the comparison of the two methods. Two values are given for the method of reference 21. The first value is that given in reference 21. The second value is calculated using the method of reference 21 and the VAR schedule assumed here. The results of application of the method of this thesis are also given. Three cases are presented: (1) Lines 1-6 with peak load of Table 8; (2) Lines with 1.1 times peak; (3) Lines 1-6 and 8 with 1.1 times peak.

A study of these two methods reveals a number of significant differences between the two methods. First, the reliability figures for Bus 2 imply that using the method of reference 21, bus 2 is completely reliable. The conditional probability with load method calculates Bus 2 to be highly reliable, but not perfectly reliable. This is due to a minor difference in the definition of reliability of disconnected buses. Reference 21 assumes that if a portion of the network is disconnected but contains the required generation and transmission within the disconnected region, the buses are reliable. The method of this thesis assumes that disconnected buses represent a stability problem and are therefore not operating acceptably. However, this difference is not a computational difference, and either method could change the treatment of disconnected buses without significant change in the computational procedures.

Second, it can be observed that all methods for all three cases rank the buses in the same order of reliability. One exception to this statement is the ranking of Buses 4 and 5 in Case One. These buses have essentially the same reliability; therefore, the difference in ranking is not significant.

Third, one of the major contributions of this thesis was not evaluated in this comparison. The line flow sensitivity data was not used due to the fact that the lines of the network did not overload. This is not representative of actual network operation, however, it does allow the comparison to be made on the basis of voltage level only.

Fourth, the major computational difference between the methods involves the load flow calculations. The method of reference 21 divides the load duration curve into 10 equal steps. A discrete load flow calculation is then performed for each of the 10 steps. This requires 10 load flow calculations for each contingency. If a load bus fails at any of the increasing load levels, the probability of unreliability is taken as the average of the probability value of the load level at which the load bus failed and the previous lower level. This 10 step feature limits the error to 50%. That is, if the failure occurs at a load slightly greater than the 10% level, the method of reference 21 would calculate this as the 5% point. This represents an error of 50% and is the worst case. This can be considered as a quantization error.

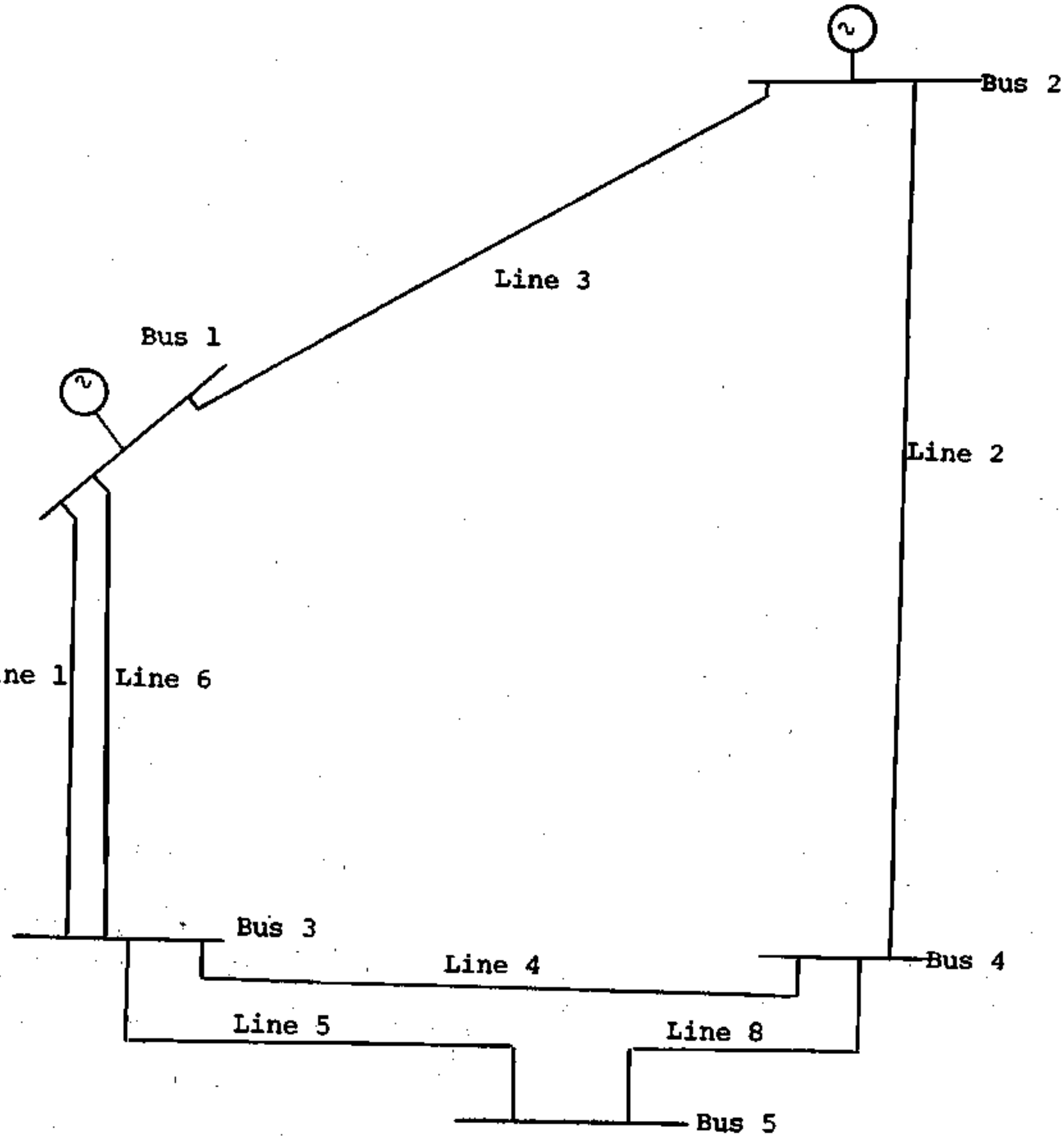

Figure 6. Five bus network.

Table 5. Generation and Load Data for the Five Bus Network

Bus	Peak Load (MW)	Power Factor	Generation Allotted Under Peak Load	Var Limits (MVAR)	Voltage Limits Max.	Min.
1	0	1.0	Swing	-10 to +10	1.05	0.97
2	20	1.0	110	0 to 40	1.05	0.97
3	85	1.0			1.05	0.97
4	40	1.0			1.05	0.97
5	10	1.0			1.05	0.97

Table 6. Line Data for the Five Bus Network.

Line	Length	Impedance (pu)	Failure Rate (/year)	Probability of Failure
1,6	30	0.0342 + j0.1800	1.5	0.001713
2	100	0.1140 + j0.6000	5.0	0.005710
3	80	0.0912 + j0.4800	4.0	0.004568
4,5,8	20	0.0228 + j0.1200	1.0	0.001142

Table 7. Reliability Indices for Five Bus Network

	Bus 2		Bus 3		Bus 4		Bus 5	
	Prob.	Freq.	Prob.	Freq.	Prob.	Freq.	Prob.	Freq.
Case 1: Lines 1-6								
Reference 21	0.0	0.0	.000875	0.8040	.001667	1.5128	.002013	1.8315
Reference 21 using this data	0.0	0.0	.00086	0.7500	.00211	1.850	.00200	1.750
Conditional Prob. with load	.000026	0.455	.0007321	0.6519	.001548	1.365	.002029	1.788
Case 2: Lines 1-6, 1.1 x Peak Load								
Reference 21	0.0	0.0	.001439	1.3070	.001669	1.5164	.002576	2.3334
Reference 21 using this data	0.0	0.0	.00120	1.050	.00360	3.150	.00264	2.30
Conditional Prob. with load	.000026	.0455	.001180	1.044	.00367	3.230	.00253	2.229
Case 3: Lines 1-6 and 8, 1.1 x Peak Load								
Reference 21	0.0	0.0	.000877	0.8095	.001440	1.3118	.000881	0.8165
Reference 21 using this data	0.0	0.0	.00120	1.050	.002284	2.000	.001713	1.500
Conditional Prob. with load	.000026	.0455	.001165	1.031	.002602	2.302	.001780	1.570

The method of this thesis uses a stochastic load flow calculation and therefore, the errors due to quantization are eliminated. However, errors due to the linearization process can occur. In order to evaluate the effects of linearization error, limits are placed on the range of linearization and additional calculations performed. The indices are calculated with limits of linearization placed at 100%, 16%, 8%, and 4% of the peak. The results of this study are shown in Table 8. It can be observed that very little change can be detected in the indices when tighter limits are placed on the linearization procedure. It can be concluded from the values in Table 11 that for this network, the errors introduced by the linearization are less than 3%.

The main conclusion which can be reached from the comparison with the Five Bus Network is that, when compared to the method of reference 21, the method presented in this thesis is more accurate and requires less computer time to perform the calculations.

Georgia Power 500-230 kV Network

In order to evaluate the method developed on a realistic network, the Georgia Power Company 500-230 kV network is used as a test case. This network is used since the network data are available and since this data represents an actual power transmission network. A map of this network is shown in Figure 7. The bus and line data for this network are presented in Appendix II. In addition to bus and line data, reliability data for this network are also required. The reliability data are calculated from the results of the study of Georgia Power outages. Table 1 lists the results of this study.

Table 8. Reliability Indices Calculated with Limits on Region of Linearization

Range of Linearization	Bus 2 Prob.	Bus 2 Error	Bus 3 Prob.	Bus 3 Error	Bus 4 Prob.	Bus 4 Error	Bus 5 Prob.	Bus 5 Error
Case 1: Lines 1-6								
100%	.000026	0	.000732	0.1%	.00154	1.9%	.002029	0.1%
16%	.000026	0	.000716	2.0%	.00155	1.3%	.002025	0.3%
8%	.000026	0	.000732	0.1%	.00157	0	.002025	0.3%
4%	.000026	0	.000731	0	.00157	0	.002031	0
Case 2: Lines 1-6, 1.1 x Peak Load								
100%	.000026	0	.00118	0	.00367	0.3%	.002531	2.8%
16%	.000026	0	.00118	0	.00370	0.5%	.002589	0.7%
8%	.000026	0	.00118	0	.00367	0.3%	.002606	0
4%	.000026	0	.00118	0	.00368	0	.002606	0
Case 3: Lines 1-6 and 8, 1.1 x Peak Load								
100%	.000026	0	.001165	0	.002602	0.8%	.001780	1.9%
16%	.000026	0	.001165	0	.002612	0.4%	.001783	1.7%
8%	.000026	0	.001165	0	.002627	0.2%	.001814	0
4%	.000026	0	.001165	0	.002622	0	.001814	0

Table 9. Reliability Data Used for Georgia Power Network

	Probability of Failure	Repairs/Day
Bus	.0000014	1.0285
500 kV line	.0000608/mile	1.846
230 kV line	.0000051/mile	2.057
115 kV line	.0000039/mile	9.057

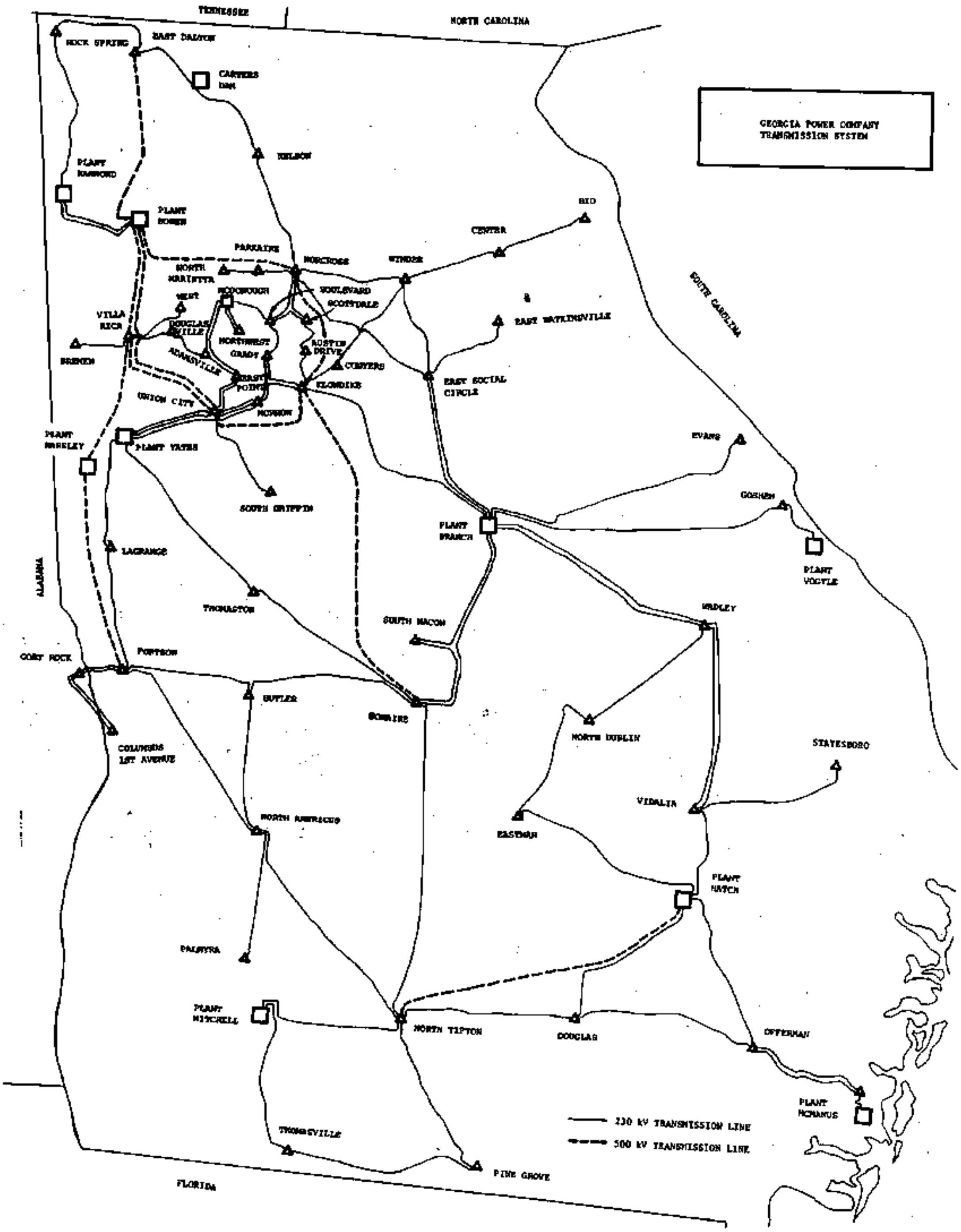

Figure 7. Georgia Power 500-230 kV Network

The reliability data for the 500 kV lines are taken from the data gathered by the Mid-America Interpool Network.[34] The reliability data are listed in Table 9.

In order to have a basis for evaluation of the results of the calculation, a Monte Carlo simulation of the Georgia Power Network was performed. The study simulated $.21\times10^{5}$ days of operation of the network.

A Gaussian two state load model is used for the simulation. The mean and variance of the low load state are 0.499 and 0.1176. The peak load state has a mean and variance of 0.629 and 0.132. These values are taken from a curve fitting procedure using the Georgia Power Company load duration curve shown in Figure 8.

The failure and repair of the network elements are modeled to have exponential time-to-failure and time-to-repair. That is, the probability that an element of the network is continuing to operate at time $t = T_1$ given that the element was operating at time $t = 0$ is:

$$P_o = e^{-\lambda T_1} \tag{62}$$

The quantity λ is called the "failure rate" of the element. Repairs of network elements are handled in the same manner. The probability that a failed element is still failed at time $t = t_2$ given that the element was failed at time $t = 0$ is:

$$P_F = e^{-\mu T_2} \tag{63}$$

where μ is the "repair rate" of the element. The value of λ can be

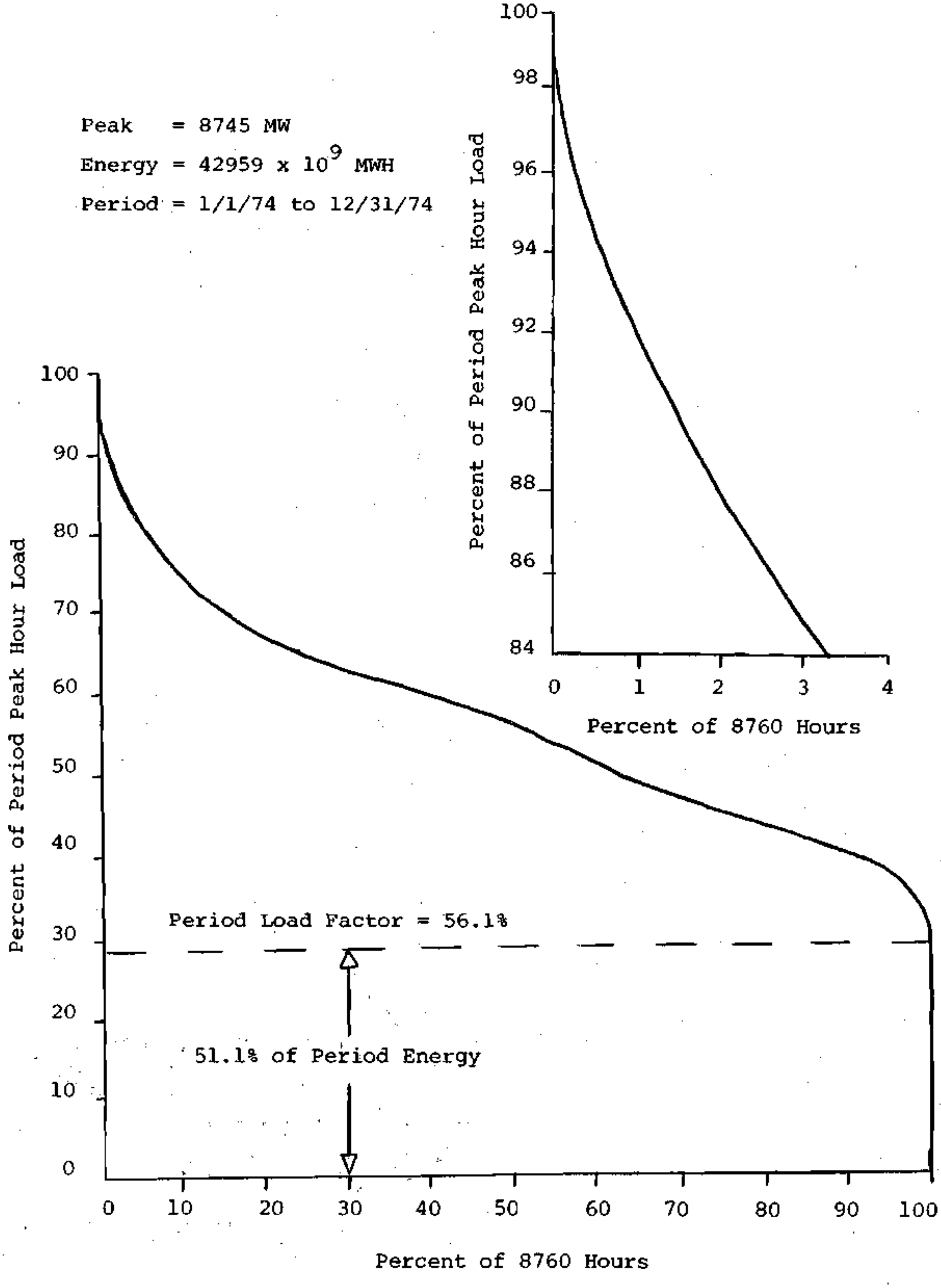

Figure 8. Georgia Power Company Load Duration

calculated from the probability of failure and repair rate given in Table 2 as:

$$\lambda = \frac{\mu p}{1 - p} \tag{64}$$

where p is the probability of failure for the element.

Since network unreliability is caused primarily by overloaded lines, the comparison is made on a line basis as well as a bus basis. That is, each line is given two indices representing the probability and frequency of failure. Using these values, the lines are then ranked with the least reliable first. The comparison is then made using the relative rankings of the least reliable lines. A comparison is also made based on the least reliable buses. The bus reliability indices are calculated based on bus voltage and line overloads using the line flow sensitivity factors for the peak load condition.

Since only finite simulation time is available, the simulation makes some errors. That is, some line outages are not analyzed while other outages are analyzed more often than necessary. For any particular contingency to be observed with a probability greater than .7, $e^{-\lambda T}$ must be less than 0.3. This implies

$$\lambda T > 1.2. \tag{65}$$

Since $T = .21\text{x}10^{5}$

$$\lambda > .57\text{x}10^{-4}. \tag{66}$$

The average repair rate is 2.0 repairs/day. Equation 64 can be rewritten as

$$P = \frac{\lambda}{\lambda + \mu} \tag{67}$$

From equations 66 and 67 it follows that contingencies with $P > .29\times10^{-4}$ will be considered with probability greater than .7.

The 25 least reliable lines as calculated by each method are shown in Table 10. These lines correspond approximately to lines with overloads caused by outages with $P > .3\times10^{-4}$. The probabilities and frequencies of overload calculated by each method are also shown in Table 10. These 25 lines are approximately 20% of the total lines in the network. There are 17 lines common to both lists. This corresponds to a correlation of 17/25 = 0.68 or approximately 0.7 as calculated above.

The 45 least reliable buses as calculated by each method are shown in Tables 11 and 12. These buses represent approximately 50% of the total buses in the system. One important correlation between the methods is that the four least reliable buses calculated by each method are the same. The range of finite values for the simulation probability index is 0.2×10^{-1} - 0.7×10^{-6}. There are 28 buses with simulation probability indices in the range 0.1×10^{-2}. This represents a very narrow portion of the available range and therefore very poor selectivity for the simulation method. However, comparisons can be made based on treating this group of buses as a unit. That is, the 14 least reliable buses can be used as a basis for comparison or the

42 least reliable buses can be used. There are 10 common buses within the 14 least reliable buses calculated by each method. This represents a correlation of 10/14 = 0.71. There are 31 buses common to the 42 least reliable buses calculated by each method for a correlation of 32/42 = 0.76. Both of these comparisons represent acceptable degrees of comparison. If a number N such that $14<N<42$ were chosen to be used as the basis for comparison, much less agreement would be calculated. However, this would not be a valid basis for comparison due to the lack of selectivity of the simulation method in this region. Since a single overloaded line can affect the reliability indices of several buses, discrepancies between the two methods tend to become distorted when comparing bus indices.

The study of the Georgia Power 500-230 kV network demonstrates that the method presented here can be applied to large networks. This is a feature not existing in previous forms of the conditional probability approach. This comparison also demonstrates that the method presented here calculates indices which agree with indices calculated using a Monte Carlo simulation within the error bounds of the simulation. That is, although the indices do not compare exactly, errors exist in the simulation results due to finite computer time. The indices calculated by the method presented here are within the error bounds established for the Monte Carlo simulation. It can therefore be concluded that the method presented here can calculate reliability indices with no greater error than a Monte Carlo simulation while using less computer time in the process.

Table 10. Georgia Power 500-230 kV Network:
25 Least Reliable Lines

Simulation			Conditional Probability with Load		
Line Number	Probability	Frequency*	Line Number	Probability	Frequency*
76	.63E-2	.24E-1	89	.18E-2	.42E-2
78	.61E-2	.24E-1	31	.78E-3	.18E-2
85	.59E-2	.22E-1	29	.41E-3	.12E-2
45	.35E-2	.13E-1	10	.29E-3	.11E-2
83	.35E-2	.13E-1	83	.27E-3	.69E-3
10	.18E-2	.65E-2	7	.14E-3	.53E-3
31	.17E-2	.60E-2	42	.14E-3	.41E-3
89	.17E-2	.49E-2	96	.11E-3	.32E-3
42	.15E-2	.62E-2	33	.11E-3	.26E-3
96	.12E-2	.51E-2	76	.87E-4	.22E-3
95	.12E-2	.50E-2	95	.84E-4	.21E-3
91	.10E-2	.41E-2	82	.84E-4	.21E-3
29	.10E-2	.36E-2	112	.79E-4	.30E-3
43	.86E-3	.32E-2	78	.79E-4	.19E-3
7	.60E-3	.22E-2	41	.72E-4	.28E-3
65	.56E-3	.20E-2	57	.71E-4	.16E-3
41	.54E-3	.21E-2	55	.71E-4	.16E-3
109	.43E-3	.16E-2	109	.70E-4	.27E-3
52	.41E-3	.23E-2	43	.69E-4	.27E-3
79	.40E-3	.15E-2	53	.41E-4	.91E-4
112	.38E-3	.13E-2	68	.37E-4	.15E-3
73	.35E-3	.11E-2	67	.37E-4	.15E-3
53	.32E-3	.16E-2	71	.20E-4	.80E-4
22	.26E-3	.85E-3	45	.16E-4	.63E-4
17	.26E-3	.80E-3	85	.96E-5	.37E-4

*Frequency is in occurrences/day

Table 11. Georgia Power 500-230 kV Network: 45 Least Reliable Buses (Simulation)

Bus	Probability	Frequency
135	.15E-1	.61E-1
1094	.14E-1	.55E-1
1096	.14E-1	.55E-1
1083	.14E-1	.54E-1
103	.72E-2	.26E-1
94	.68E-2	.26E-1
92	.68E-2	.26E-1
90	.65E-2	.25E-1
10	.63E-2	.25E-1
99	.63E-2	.22E-1
102	.62E-2	.22E-1
104	.59E-2	.21E-1
105	.59E-2	.21E-1
98	.58E-2	.20E-1
60	.47E-2	.19E-1
56	.47E-2	.18E-1
59	.46E-2	.18E-1
626	.44E-2	.15E-1
1033	.43E-2	.18E-1
55	.43E-2	.16E-1
1034	.42E-2	.17E-1
57	.42E-2	.16E-1
87	.33E-2	.12E-1
44	.28E-2	.93E-2
89	.27E-2	.94E-2
2	.25E-2	.10E-1
43	.19E-2	.67E-2
17	.10E-2	.68E-2
15	.18E-2	.66E-2
16	.18E-2	.66E-2
53	.16E-2	.63E-2
95	.14E-2	.42E-2
111	.12E-2	.50E-2
40	.11E-2	.41E-2
31	.11E-2	.33E-2
280	.10E-2	.43E-2
281	.10E-2	.43E-2
41	.10E-2	.38E-2
34	.10E-2	.38E-2
39	.10E-2	.38E-2
50	.10E-2	.37E-2
209	.10E-2	.37E-2
58	.91E-3	.34E-2
1272	.91E-3	.34E-2
73	.83E-3	.32E-2
77	.69E-3	.24E-2

Table 12. Georgia Power 500-230 kV Network: 45 Least Reliable Buses (Conditional Probability)

Bus	Probability	Frequency
135	.49E-0	.99E-0
1094	.17E-1	.66E-1
1096	.13E-1	.52E-1
1083	.13E-1	.52E-1
105	.84E-2	.20E-1
99	.84E-2	.19E-1
98	.83E-2	.19E-1
104	.83E-2	.19E-1
102	.83E-2	.19E-1
103	.83E-2	.19E-1
626	.83E-2	.19E-1
15	.81E-2	.19E-1
16	.81E-2	.19E-1
17	.81E-2	.19E-1
1033	.47E-2	.18E-1
1034	.47E-2	.18E-1
2	.29E-1	.53E-2
34	.14E-2	.41E-2
281	.13E-2	.49E-2
39	.13E-2	.40E-2
280	.13E-2	.42E-2
40	.13E-2	.37E-2
41	.13E-2	.37E-2
50	.13E-2	.37E-2
209	.13E-2	.37E-2
95	.10E-2	.28E-2
44	.92E-3	.24E-2
31	.85E-3	.22E-2
72	.46E-3	.19E-2
67	.37E-3	.15E-2
1042	.35E-3	.14E-2
63	.34E-3	.14E-2
53	.34E-3	.13E-2
1272	.31E-3	.13E-2
58	.26E-3	.11E-2
60	.26E-3	.10E-2
87	.23E-3	.92E-3
86	.20E-3	.82E-3
57	.17E-3	.67E-3
76	.16E-3	.63E-3
82	.16E-3	.63E-3
62	.20E-3	.79E-3
97	.16E-3	.65E-3
52	.15E-3	.60E-3
56	.15E-3	.59E-3

Computation Time

In order for this procedure to be practical, it must not require an excessive amount of computer time to compute the indices. Most of the computation time is used for performing load flow calculations. The remaining calculations require only minimal computation time. The procedure is implemented on a 32 K NOVA mini-computer which does not have an integral clock. Therefore, the analysis of computation requirements can only be approximate.

For the Three Bus Network, the calculations were performed by hand and therefore computer requirements for this case cannot be measured.

For the Five Bus Network, the computer was used, but, solution time was too small to be measured externally. For this case the relative computer time can be compared to the other method. The other method required 10 discrete load flow calculations. The method presented here required one stochastic load flow calculation. A stochastic load flow calculation involves one discrete load flow calculation plus a series of additional calculations which are roughly equivalent to an additional iteration of the discrete load flow. Since the average discrete load flow calculation for this case required approximately three iterations, the speed advantage of the method presented here over previous methods requiring multiple deterministic load flow can be calculated approximately as 10-1.3 or about 7.5 to one.

For the Georgia Power Network, a stochastic load flow calculation is required for each generator. There are 17 generators. One stochastic load flow calculation was observed to require about 20 minutes of computer time. Therefore, the complete calculation requires approximately 340 minutes or about six hours. This time can be compared with the Monte Carlo simulation which required about 24 hours. The speed advantage here is about four-to-one.

It should be emphasized again that the numbers given above are relative to the NOVA 830 computer frame. In absolute terms, the computation times will be much less with large computer implementation. The relative time advantages as compared with other procedures are, however, accurately reflected.

Summary

The method of this thesis has been shown superior to other methods in all three test cases. This method is shown to be significantly more accurate than the approximate methods used in test case one. The method of this thesis is shown to be both more accurate and less costly with computer time than the other method in test case two. These advantages result primarily from the use of the stochastic load flow algorithms. The speed of the decoupling algorithm along with the accuracy and speed of the partially coupled equations result in both increased accuracy and reduced computation time.

The first two test cases demonstrate that the method of this thesis compares very favorably with other methods when the test network is small and therefore unrealistic. These test cases are, however, necessary since the previous methods can be applied to small networks only. The major advantage of this method is demonstrated in the third test case. That is, the method can be applied to a large network using generally available data. This applicability is made possible through the use of sensitivity factors. The practicality of this method is enhanced by the efficiency of the stochastic load flow. The indices calculated by this procedure are shown to compare favorably with indices calculated using a Monte Carlo simulation of the operation of the test network.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A method of calculating indices of reliability for a power transmission system has been presented which has been demonstrated to be superior to previous methods. The primary feature of the method which makes possible its increased utility is the ability to account for the effects of line loading and bus voltages while remaining applicable to a large network. The applicability to a large network has been demonstrated by applying the method to the 90-bus Georgia Power 500-230 kV Network. In addition, in comparing the developed method with previous techniques on smaller networks where quantitative differences can be detected, the method has been shown to be superior to previously existing methods. Examination of three- and five-bus example networks has demonstrated that this method achieves a higher degree of accuracy without sacrificing efficiency. In one case the method was shown to be both more efficient and more accurate than the method used for comparison.

The objective of this thesis was the development of network reliability indices and associated computational procedures which would be useful as a tool for transmission planners if the indices possessed certain desirable characteristics. It has been demonstrated that the method possesses all of these characteristics. These characteristics are: (1) the method is efficient with computer time; (2) the method is sensitive to network parameters; (3) the indices are consistent with network operation; (4) the indices are relatively insensitive to errors in component reliability data; (5) the method considers the effects of line overloading and bus voltages; (6) the indices are bus oriented and, therefore, useful as a direct planning aid. Since all the above characteristics have been demonstrated, it can be concluded that the method will be useful to transmission planners of the future. This is in direct contrast to previous methods which have limited usefulness.

Recommendations

Although the method of this thesis represents a major advance in the field of network reliability, improvements can be made. The following suggestions are offered as possible areas of improvement.

(1) The sensitivity factors represent a very powerful tool which have not been fully exploited. The sensitivity factors could be used in more refined ways to determine network contingencies which lead to unreliable operation. This would remove the requirement to perform load flow calculations for every contingency. It is possible that the base case load flow is the only discrete load flow calculation required.

(2) As more data becomes available, more sophisticated models of both the load and the network could be developed. In particular, modeling of the load

as a multi-step or continuous process is feasible. The load model could also be improved by separating the various types of loads: commercial, industrial, and residential. Improvements in the network model should become obvious as more outage data becomes available.

(3) The method needs to be evaluated in the environment for which it was intended, i.e. the planning department of an electric utility. This evaluation will produce many suggestions for improvements.

APPENDIX I

PROOF OF CONSISTENCY

The concept of consistency was introduced in Chapter IV. To be considered consistent, the indices must exhibit three characteristics: (1) an increase in the duration of peak load produces an increase in the reliability indices; (2) an increase in the duration of network contingencies produces an increase in the reliability indices; (3) a decrease in the load or the addition of an additional line produces a decrease in the reliability indices. To avoid confusion, it should be noted here that an increase in the reliability indices represents a decrease in the reliability of the network.

Equation 54 describes the mathematical procedure used to calculate the indices:

$$P_k = \sum_{i=0}^{L} \sum_{j=0}^{M} P_{Li} P_{Nj} P_{ijk} \tag{54}$$

The following characteristics of the models are required for the proof:

$$0 \le P_{Li} \le 1 \tag{A-1}$$

$$0 \le P_{Nj} \le 1 \tag{A-2}$$

$$0 \le P_{ijk} \le 1 \tag{A-3}$$

$$\sum_{i=0}^{L} P_{Li} = 1 \tag{A-4}$$

$$\sum_{j=0}^{M} P_{Nj} = 1. \tag{A-5}$$

The following characteristics of the network are assumed:

$$P_{0jk} < P_{ijk}, \quad i \neq 0 \tag{A-6}$$

$$P_{i0k} < P_{ijk}, \quad j \neq 0 \tag{A-7}$$

Equation A-6 is a result of the fact that the mean of the peak load is larger than the mean of the low load state. Therefore, the probability of the load exceeding some specific value is greater during periods of peak load. Equation A-7 states that a line removed from a network reduces the load carrying ability of the network. Both of these assumptions are reasonable for realistic networks. In the following proofs (a) denotes one situation and (b) denotes another.

In order to prove consistency (1) equation 7 is used

$$P_{Li} = \frac{n_i e}{P}, \quad i \neq 0$$

$$P_{L0} = 1-e \tag{7}$$

If

$$e_{(a)} < e_{(b)}$$

then

$$P_{Li(a)} < P_{Li(b)}$$

$$P_{L0(1)} > P_{L0(2)}$$

Applying equation 54:

$$P_{k(a)} = \sum_{i=0}^{L} \sum_{j=0}^{M} P_{Li(a)} P_{Nj} P_{ijk}$$

$$= \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} P_{Li(b)} P_{ijk} - \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} [P_{Li(b)} - P_{Li(a)}] P_{ijk}$$

$$= P_{k(b)} - \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} [P_{Li(b)} - P_{Li(a)}] P_{ijk}$$

$$P_{k(b)} - P_{k(a)} = \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} [P_{Li(b)} - P_{Li(a)}] P_{ijk}$$

$$= \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} [P_{Li(b)} - P_{Li(a)}][P_{ijk} - P_{0jk}] + \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L}$$

$$[P_{Li(b)} - P_{Li(a)}] P_{0jk}$$

$$= \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} [P_{Li(b)} - P_{Li(a)}][P_{ijk} - P_{0jk}] + P_{0jk} \sum_{j=0}^{M} P_{Nj}$$

$$\left[\sum_{i=0}^{L} P_{Li(b)} - \sum_{i=0}^{L} P_{Li(a)}\right]$$

$$= \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} [P_{Li(b)} - P_{Li(a)}][P_{ijk} - P_{0jk}] + P_{0jk} \sum_{j=0}^{M} P_{Nj}$$

$$[1 - 1]$$

$$= \sum_{j=0}^{M} P_{Nj} \sum_{i=0}^{L} [P_{Li(b)} - P_{Li(a)}][P_{ijk} - P_{0jk}]$$

Each term in this expression is greater than zero, therefore:

$$P_{k(b)} - P_{k(a)} > 0$$

$$P_{k(a)} < P_{k(b)}$$

This proves consistency (1).

Proof of consistency (2) involves the use of the network model definition:

$$F_{Nj} = \mu_i P_{Nj}$$

$$P_{Nj} = F_{Nj}/\mu_i$$

$$= d_i F_{Nj}$$

$$d_{j(a)} < d_{j(b)} \quad j \neq 0$$

$$P_{Nj(a)} < P_{Nj(b)} \quad j \neq 0$$

$$\sum_{j=0}^{M} P_{Nj} = 1$$

$$P_{N0(a)} > P_{N0(b)}$$

$$P_{k(a)} = \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} P_{Nj(a)} P_{ijk}$$

$$= \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} P_{Nj(b)} P_{ijk} - \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}] P_{ijk}$$

$$= P_{k(b)} - \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}] P_{ijk}$$

$$P_{k(b)} - P_{k(a)} = \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}] P_{ijk}$$

$$= \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}][P_{ijk} - P_{0jk}] +$$

$$\sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}] P_{ijk}$$

$$= \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}][P_{ijk} - P_{0jk}] +$$

$$\sum_{i=0}^{L} P_{Li} P_{ijk} [\sum_{j=0}^{M} P_{Nj(b)} - \sum_{j=0}^{M} P_{Nj(a)}]$$

$$= \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}][P_{ijk} - P_{0jk}] +$$

$$\sum_{i=0}^{L} P_{Li} P_{ijk} [1 - 1]$$

$$= \sum_{i=0}^{L} P_{Li} \sum_{j=0}^{M} [P_{Nj(b)} - P_{Nj(a)}][P_{ijk} - P_{0jk}]$$

Each term in this expression is greater than zero, therefore:

$$P_{k(b)} - P_{k(a)} > 0$$

$$P_{k(a)} < P_{k(b)}$$

This proves consistency (2). Consistency (3) involves the addition of a line which improves the load carrying ability of the network and therefore,

$$P_{ijk(a)} > P_{ijk(b)}$$

The same expression results if the load is reduced.

$$P_{k(a)} = \sum_{i=0}^{L} \sum_{j=0}^{M} P_{Li} P_{Nj} P_{ijk(a)}$$

$$P_{k(b)} = \sum_{i=0}^{L} \sum_{j=0}^{M} P_{Li} P_{Nj} P_{ijk(b)}$$

If these equations are compared term by term,

$$P_{k(a)} > P_{k(b)}$$

This proves consistency (4).

APPENDIX II

GEORGIA POWER 500-230 kV NETWORK DATA

Table A-1. Georgia Power Bus Data.

Bus Number	Bus Name	Peak Load (MVA) Real	Reactive	Voltage Level (kV)	Generator Number
1	BOWEN 1+2 8	.00	.00	500	1
2	BOWEN 3+4 8	.00	.00	500	2
3	NORCROSS 8	.00	.00	500	
4	KLONDIKE 8	.00	.00	500	
5	UNION CITY 8	.00	.00	500	
10	BONAIRE 8	.00	.00	500	
15	ROCK SPRINGS 5	75.58	26.65	161	
16	ROCK SPRINGS 6	.00	.00	230	
17	E. DALTON	145.30	58.78	230	
19	BOWEN 6	.00	.00	230	
20	HAMMOND 6	.00	.00	230	3
23	BREMEN 6	30.98	38.39	230	
24	VILLA RICA 6	.00	.00	230	
26	ADAMSVILLE 6	18.20	9.18	230	
27	MCDONOUGH 6	.00	.00	230	
28	MCDONOUGH 3-T	.00	.00	115	4
29	E. POINT 6	336.38	95.93	230	
31	UNION CITY 6	16.70	6.62	230	
32	NORTHWEST 1 6	.00	.00	230	
33	NORTHWEST 2 6	.00	.00	230	
34	N. MARIETTA 6	90.35	23.74	230	
35	W. MARIETTA 6	145.21	23.54	230	
39	PARKAIRE	167.51	59.04	230	
40	NORCROSS 6	31.30	11.60	230	
41	BOULEVARD 6	318.94	66.06	230	
43	GRADY 6	278.50	55.32	230	
44	MORROW 6	272.88	72.32	230	
45	KLONDIKE 6	2.65	.95	230	
46	SNAPFINGER 6	22.80	7.38	230	

Bus Number	Bus Name	Peak Load (MVA) Real	Reactive	Voltage Level (kV)	Generator Number
47	AUSTIN DR. 6	56.54	28.57	230	
48	DURHAM PARK 6	27.20	11.10	230	
50	SCOTTDALE 5	186.31	84.00	230	
52	CARTERS DAM 6	11.30	7.19	230	5
53	NELSON 6	80.82	15.50	230	
55	CONYERS 6	91.49	36.94	230	
56	WINDER 6	138.62	34.99	230	
57	CENTER 6	166.05	22.86	230	
58	BIO 6	118.86	7.58	230	
59	SOCIAL CIRCLE	93.51	25.29	230	
60	E. WATKINSVILLE	75.10	28.0	230	
62	WARRENTON 6	.00	.00	230	
63	EVANS 6	143.32	19.13	230	
67	GOSHEN 6	166.42	35.87	230	
70	BRANCH 1-2 6	.00	.00	230	6
71	BRANCH 3-4 6	.00	.00	230	7
72	VOGTLE 6T	.00	.00	230	8
73	WADLEY 6	127.10	34.06	230	
74	DUBLIN 6	57.78	8.43	230	
75	EASTMAN 6	12.70	7.60	230	
76	HATCH 6	18.30	10.00	230	
77	VIDALIA 6	142.12	6.76	230	
81	BRUNSWICK 6	58.17	1.11	230	
82	OFFERMAN 6	82.45	11.73	230	
83	MCMANUS 6	44.66	- 6.16	230	9
86	DOUGLAS 6	71.76	20.36	230	
87	PINE GROVE	85.84	17.16	230	
89	TIFTON 6	52.20	37.00	230	
90	BONAIRE 6	294.74	52.46	230	
92	MACON 6	236.44	44.92	230	
94	THOMASTON 6	120.38	29.48	230	

Bus Number	Bus Name	Peak Load (MVA) Real	Reactive	Voltage Level (kV)	Generator Number
95	GRIFFEN 6	116.90	22.45	230	
96	YATES 6	.00	.00	230	10
97	LAGRANGE 6	114.10	32.70	230	
98	GOATROCK 6	99.96	- 25.67	230	
99	COLUMBUS 6	131.70	35.15	230	
102	FORTSON 6	80.98	5.62	230	
103	BUTLER 6	21.60	12.50	230	
104	N. AMERICUS 6	114.46	5.23	230	
105	PALMYRA 6	65.50	- 1.98	230	
111	EATONTON 6	14.10	7.47	230	
116	DOUGLASVILLE 6	130.25	19.20	230	
135	HAMMOND 3	436.00	32.20	115	11
185	NORTHWEST 3	315.00	76.00	115	
209	NORCROSS	370.43	101.50	115	
280	BUFORD 3	44.00	28.80	115	
281	BUFORD 3	76.10	23.18	115	12
626	GASTON 6	.00	.00	230	13
1025	WIDOWS CREEK 1	633.59	111.67	161	14
1026	WIDOWS CREEK 23	.00	.00	230	
1027	WIDOWS CREEK 50	65.86	-114.49	500	
1033	BULL RUN	557.88	290.32	161	15
1034	BULL RUN 500-kV	136.59	- 98.40	500	
1036	NICKAJACK	108.49	46.71	161	16
1042	OGLETHORPE	252.01	28.34	161	
1044	WIDOWS CREEK	391.35	60.50	161	SLACK
1073	SEQUOYA 500-kV	136.95	- 7.71	500	
1083	PARADISE 500-kV	711.72	- 39.00	500	17
1094	CUMBERLAND 8	55.52	.54	500	
1096	WILSON 8	171.64	- 93.88	500	
1272	HARTWELL DAM 6	- 86.70	7.28	230	

Table A-2. Georgia Power Generator Data.

Number	Bus	Start Number	Number of Units	a_o	a_1	a_2	Real Power Max	Real Power Min	Reactive Power Max	Reactive Power Min
1	1	1	4	300.0	1.10	.00070	1410.	150.	1120.	-300.
2	2	14	2	300.0	1.10	.00070	1410.	150.	1120.	-300.
3	20	13	1	305.0	1.12	.00074	520.	100.	322.	-300.
4	28	2	1	305.0	1.12	.00074	520.	100.	312.	-300.
5	52	15	1	5.0	0.010	.10E-5	250.	80.	100.	-300.
6	70	4	2	300.0	1.10	.00070	700.	150.	755.	-300.
7	71	8	2	300.0	1.10	.00070	700.	150.	755.	-300.
8	72	18	1	100.0	1.20	.010	328.	80.	150.	-300.
9	83	6	1	305.0	1.12	.00074	316.	80.	100.	-300.
10	96	12	1	300.0	1.10	.00070	1011.	300.	428.	-300.
11	135	9	1	305.0	1.12	.00074	317.	80.	163.	-300.
12	281	5	1	8.0	0.013	.13E-5	105.	50.	52.	-300.
13	626	17	2	300.0	1.10	.00070	689.	100.	500.	-300.
14	1025	3	2	305.0	1.12	.00072	767.	200.	280.	-495.
15	1033	10	2	305.0	1.12	.00070	875.	200.	334.	-485.
16	1036	16	1	9.0	0.014	.14E-5	64.	25.	500.	- 15.
17	1083	7	1	300.0	1.10	.00070	1070.	250.	421.	-633.
18	1272	11	1	9.5	0.015	.15E-5	240.	80.	100.	-300.

Table A-3. Georgia Power Line Data.

Line Number	From Bus	To Bus	Resistance (%)*	Inductance (%)*	Line Charging (MVA)	Tap	Capacity (MVA)
1	2	1	.000	1.000	.000	.000	9999
2	3	1	.060	.970	89.800	.000	2800
3	4	3	.030	.560	52.110	.000	2800
4	5	1	.070	1.220	113.500	.000	2800
5	5	4	.050	.790	73.150	.000	2800
6	10	4	.120	1.900	177.000	.000	2800
7	16	15	.000	2.540	.000	1.000	336
8	17	16	.420	3.980	8.200	.000	306
9	1	19	.000	1.200	.000	1.020	900
10	20	16	.700	6.660	13.700	.000	306
11	20	19	.460	4.360	8.900	.000	586
12	20	19	.460	4.360	8.900	.000	586
13	24	19	.180	2.860	11.900	.000	1200
14	24	23	.300	2.880	5.900	.000	586
15	27	26	.100	.900	1.900	.000	306
16	27	28	.000	1.520	.000	1.050	672
17	29	26	.100	.900	1.900	.000	306
18	29	27	.190	1.790	3.700	.000	306
19	5	31	.000	.900	.000	.980	1344
20	31	24	.180	3.010	12.500	.000	1200
21	31	29	.140	1.570	3.300	.000	332
22	31	29	.140	1.570	3.300	.000	332
23	32	27	.070	.670	1.400	.000	306
24	33	27	.070	.670	1.400	.000	306
25	35	24	.290	2.790	5.720	.000	586
26	39	34	.130	1.210	2.500	.000	584
27	3	40	.000	.900	.000	.980	1344
28	40	39	.170	1.560	3.200	.000	584
29	41	27	.160	1.280	3.700	.000	306
30	41	40	.200	1.770	4.100	.000	586

Line Number	From Bus	To Bus	Resistance (%)*	Inductance (%)*	Line Charging (MVA)	Tap	Capacity (MVA)
31	44	31	.230	1.720	3.400	.000	247
32	44	31	.180	1.680	3.400	.000	304
33	44	43	.230	1.460	2.640	.000	320
34	45	43	.250	2.390	4.800	.000	306
35	46	45	.150	1.340	3.000	.000	586
36	47	46	.050	.390	.800	.000	586
37	48	47	.090	.790	1.800	.000	586
38	50	40	.200	1.780	4.100	.000	586
39	50	41	.160	1.410	3.300	.000	586
40	50	48	.090	.780	1.800	.000	586
41	52	17	.370	2.810	5.500	.000	258
42	53	40	.680	5.140	10.100	.000	258
43	53	52	.560	4.200	8.200	.000	258
44	55	44	.540	3.350	6.260	.000	220
45	56	40	.550	4.160	8.200	.000	220
46	56	55	.670	4.120	7.800	.000	220
47	57	56	.540	3.330	6.300	.000	426
48	58	57	.760	4.670	8.800	.000	426
49	59	56	.370	3.490	7.100	.000	584
50	60	59	.440	3.330	6.500	.000	498
51	63	62	.480	4.540	9.320	.000	584
52	70	40	1.180	11.200	23.000	.000	306
53	70	45	.980	9.290	19.000	.000	306
54	70	62	.630	5.910	12.130	.000	584
55	70	67	1.190	11.280	23.100	.000	306
56	71	70	.000	10.000	.000	.000	9999
57	72	67	.300	2.850	5.840	.000	586
58	73	70	.870	8.230	16.900	.000	584
59	73	70	.870	8.230	16.900	.000	584
60	74	73	.740	5.570	10.930	.000	498
61	75	74	.410	3.880	7.970	.000	586

Line Number	From Bus	To Bus	Resistance (%)*	Inductance (%)*	Line Charging (MVA)	Tap	Capacity (MVA)
62	76	75	.840	7.910	16.200	.000	586
63	77	73	.900	6.770	13.300	.000	258
64	77	73	.900	6.790	13.300	.000	258
65	77	76	.450	3.380	6.620	.000	247
66	82	76	.740	5.550	10.900	.000	247
67	82	81	.760	5.680	11.200	.000	258
68	82	81	.760	5.700	11.200	.000	258
69	83	81	.100	.890	1.800	.000	586
70	86	76	.700	6.650	13.630	.000	586
71	86	82	1.140	7.020	13.300	.000	220
72	89	76	.500	8.190	34.120	.000	1200
73	89	86	1.000	6.130	11.600	.000	220
74	89	87	.920	6.870	13.500	.000	258
75	10	90	.000	1.200	.000	.980	1344
76	90	70	.840	7.930	16.300	.000	304
77	90	89	1.850	11.350	21.500	.000	220
78	92	70	.650	6.180	12.700	.000	304
79	92	90	.530	4.980	10.200	.000	304
80	94	90	.990	7.410	14.500	.000	257
81	95	31	.650	4.840	9.500	.000	498
82	96	31	.370	3.460	7.100	.000	304
83	96	31	.470	3.540	6.900	.000	247
84	96	44	.550	5.180	10.620	.000	584
85	96	94	1.050	7.850	15.400	.000	258
86	97	96	.650	4.840	9.600	.000	496
87	99	98	.250	1.600	3.000	.000	220
88	102	98	.240	1.460	2.800	.000	220
89	103	90	1.060	6.510	12.300	.000	220
90	103	102	.940	5.730	10.800	.000	220
91	104	89	1.460	8.950	16.900	.000	220
92	104	98	1.630	10.030	19.000	.000	220

Line Number	From Bus	To Bus	Resistance (%)*	Inductance (%)*	Line Charging (MVA)	Tap	Capacity (MVA)
93	104	103	.740	4.720	9.900	.000	224
94	105	104	.650	4.890	9.600	.000	258
95	111	59	.460	4.400	9.100	.000	304
96	111	70	.150	1.400	2.800	.000	304
97	116	24	.150	1.400	2.900	.000	586
98	116	26	.290	2.370	5.030	.000	498
99	20	135	.000	3.040	.000	1.100	336
100	32	185	.000	3.470	.000	1.000	280
101	33	185	.000	3.480	.000	1.000	280
102	40	209	.000	3.600	.000	1.000	280
103	40	209	.000	3.600	.000	1.000	280
104	280	209	1.910	5.790	2.300	.000	110
105	281	280	.600	2.840	.400	.000	95
106	626	96	1.380	13.710	28.387	.000	581
107	626	98	1.360	13.490	27.971	.000	581
108	626	98	1.360	13.490	27.971	.000	581
109	1026	17	.890	8.500	16.900	.000	304
110	1034	1033	.000	1.880	.000	1.020	9999
111	1036	1025	.710	4.160	1.980	.000	9999
112	1042	15	.360	3.140	1.650	.000	390
113	1042	1025	1.520	9.100	4.320	.000	9999
114	1042	1036	1.630	8.820	3.960	.000	9999
115	1044	1025	.000	1.000	.000	.000	9999
116	1026	1044	.000	1.200	.000	.975	9999
117	1027	1044	.000	1.500	.000	.929	9999
118	1044	1042	1.340	8.760	4.540	.000	9999
119	1044	1042	1.340	8.760	4.540	.000	9999
120	1073	1	.120	1.860	167.670	.000	2800
121	1073	1027	.080	1.100	92.120	.000	9999
122	1073	1034	.130	1.830	153.760	.000	9999
123	1094	1083	.800	10.650	.000	.000	9999

Line Number	From Bus	To Bus	Resistance (%)*	Inductance (%)*	Line Charging (MVA)	Tap	Capacity (MVA)
124	1096	1034	.230	3.380	290.020	.000	9999
125	1096	1083	.810	13.550	.000	.000	9999
126	1096	1094	.460	7.270	.000	.000	9999
127	1272	58	.220	1.380	2.600	.000	420

*Per Cent Impedance on 100 MVA base.

APPENDIX III

CALCULATION OF INDICES FOR THREE BUS NETWORK

The data for this network is presented in Chapter V. In addition to the data provided, the following assumptions were used:

(1) 100 MVA base for per unit system.

(2) Impedance of line 4 is 1%.

(3) Lines have no resistive component of impedance.

(4) σ^2 of load is zero. This assumption is used so that the methods will be compared for the same load model.

(5) Probability of bus failure is the probability that two or more lines connected to that bus fail.

(6) Buses are assumed within voltage limits.

Assumption 2 leads to the following line impedances

Line	Zp.u. equal impedance	Zp.u. impedance proportional to P_F
1 or 2	j.01	j.00833
3	j.01	j.00167
4	j.01	j.01

The bus failure probabilities can be calculated using assumption 5.

Bus	P_F
B	$.256\times10^{-6}$
C	$.440\times10^{-7}$

EQUAL IMPEDANCE

$$B' = B'' = \begin{bmatrix} -300 & 100 \\ 100 & -200 \end{bmatrix}$$

$$[B']^{-1} = \begin{bmatrix} -.004 & -.002 \\ -.002 & -.006 \end{bmatrix}$$

PROPORTIONAL IMPEDANCE

$$B' = B'' = \begin{bmatrix} -839 & 599 \\ 599 & -699 \end{bmatrix}$$

$$[B']^{-1} = \begin{bmatrix} -.00307 & -.00263 \\ -.00263 & -.00369 \end{bmatrix}$$

Sensitivity Factors

Equal impedance:

$$S_{1B} = -[-100(0-(-.004))]$$
$$= .4$$

$$S_{2B} = S_{1B} = .4$$

$$S_{3B} = -[-100(-.004-(-.002))]$$
$$= -.2$$

$$S_{4B} = -[-100(0-(-.002))]$$
$$= .2$$

Proportional impedance:

$$S_{1B} = -[-120(0-(-.00307))]$$
$$= .364$$

$$S_{2B} = S_{1B} = .364$$

$$S_{3B} = -[-599(-.00307-(-.00263))]$$
$$= -.263$$

$$S_{4B} = -[-100(0.-(-.00263))]$$
$$= .263$$

EQUAL IMPEDANCE

$S_{1C} = -[-100(0-(-.002))]$

$= .2$

$S_{2C} = S_{1C} = .2$

$S_{3C} = -[-100(-.002-(-.006))]$

$= .4$

$S_{4C} = -[-100(0-(-.006))]$

$= .6$

PROPORTIONAL IMPEDANCE

$S_{1C} = -[-120(0.-(-.00263))]$

$= .315$

$S_{2C} = S_{1C} = .315$

$S_{3C} = -[-599(-.00263-(-.00369))]$

$= .635$

$S_{4C} = -[-100(0-(-.00369))]$

$= .369$

Network State - 0

All Lines In

Line	Flow	Overload Point 80 MW	100 MW
1 or 2	.6xpeak	133	167
3	.2xpeak	400	500
4	.8xpeak	100	125

Flow	Overload Point 80 MW	100 MW
.68xpeak	117	147
.37xpeak	216	270
.63xpeak	128	158

All overload points are greater than all peak loads.

EQUAL IMPEDANCE

PROPORTIONAL IMPEDANCE

$$P_{i0B} = P_{i0C} = 0 = F_{i0B} = F_{i0C}$$

Network State - 1

Line 1 Out

Equal impedance:

Line	Flow	Overload Point 80 MW	Overload Point 100 MW
2	1.xpeak	80	100
3	0	0	0
4	1.xpeak	80	100

Proportional impedance:

Flow	Overload Point 80 MW	Overload Point 100 MW
1.08xpeak	74	92
.083xpeak	963	1204
.917xpeak	87	109

Equal impedance:

Load State	$P_{i1B}=P_{i1C}$ 80 MW	$P_{i1B}=P_{i1C}$ 100 MW
0	0	0
1	0	0
2	0	0
3	0	0
4	1	0
5	1	0

Proportional impedance:

$P_{i1B}=P_{i1C}$ 80 MW	$P_{i1B}=P_{i1C}$ 100 MW
0	0
0	0
0	0
1	0
1	0
1	1

Network State - 2

Line 2 Out

See Network State 1

Network State - 3

Line 3 Out

EQUAL IMPEDANCE

Line	Flow	Overload Point 80 MW	Overload Point 100 MW
1 or 2	.5xpeak	160	200
4	1.xpeak	80	100

Load State	$P_{i3B}=P_{i3C}$ 80 MW	$P_{i3B}=P_{i3C}$ 100 MW
0	0	0
1	0	0
2	0	0
3	0	0
4	1	0
5	1	0

PROPORTIONAL IMPEDANCE

Flow	Overload Point 80 MW	Overload Point 100 MW
.5xpeak	160	200
1.xpeak	80	100

$P_{i3B}=P_{i3C}$ 80 MW	$P_{i3B}=P_{i3C}$ 100 MW
0	0
0	0
0	0
0	0
1	0
1	0

Network State - 4

Line 4 Out

EQUAL IMPEDANCE

Line	Flow	Overload Point 80 MW	Overload Point 100 MW
1 or 2	1.xpeak	80	100
3	1.xpeak	80	100

PROPORTIONAL IMPEDANCE

Flow	Overload Point 80 MW	Overload Point 100 MW
1.xpeak	80	100
1.xpeak	80	100

EQUAL IMPEDANCE

Load State	$P_{i4B}=P_{i4C}$ 80 MW	100 MW
0	0	0
1	0	0
2	0	0
3	0	0
4	1	0
5	1	0

PROPORTIONAL IMPEDANCE

$P_{i4B}=P_{i4C}$ 80 MW	100 MW
0	0
0	0
0	0
0	0
1	0
1	0

Network State - 5

Bus B Out

$P_{i5B}=1$

Load State	P_{i5C} 80 MW	100 MW
0	0	0
1	0	0
2	0	0
3	0	0
4	1	0
5	1	0

P_{i5C} 80 MW	100 MW
0	0
0	0
0	0
0	0
1	0
1	0

EQUAL IMPEDANCE

PROPORTIONAL IMPEDANCE

Network State - 6

Bus C Out

$P_{i6C}=1$

$P_{i6B}=1$

For 80 MW Line Capacity

Equal impedance:

$$P_B = \sum_{j=0}^{6} \sum_{i=0}^{5} P_{Li} P_{Nj} P_{ijB}$$

$$= (.428\times10^{-3})(.1+.1)$$
$$+ (.428\times10^{-3})(.1+.1)$$
$$+ (.856\times10^{-4})(.1+.1)$$
$$+ (.5136\times10^{-3})(.1+.1)$$
$$+ (.256\times10^{-6})(1)$$
$$= .291\times10^{-3}$$

Proportional impedance:

$$P_B = (.428\times10^{-3})(.1+.1+.1)$$
$$+ (.428\times10^{-3})(.1+.1+.1)$$
$$+ (.856\times10^{-4})(.1+.1)$$
$$+ (.5136\times10^{-3})(.1+.1)$$
$$+ (.256\times10^{-6})(1)$$
$$= .377\times10^{-3}$$

EQUAL IMPEDANCE

$$P_C = \sum_{j=0}^{6} \sum_{i=0}^{5} P_{Li} P_{Nj} P_{ijc}$$

$$= (.428\text{x}10^{-3})(.1+.1)$$
$$+ (.428\text{x}10^{-3})(.1+.1)$$
$$+ (.856\text{x}10^{-4})(.1+.1)$$
$$+ (.5136\text{x}10^{-3})(.1+.1)$$
$$+ (.256\text{x}10^{-6})(.1+.1)$$
$$+ (.44\text{x}10^{-7})(1)$$
$$= .291\text{x}10^{-3}$$

$$P_B = \sum_{j=0}^{4} \sum_{i=0}^{5} P_{Li} P_{Nj} \left(\frac{1}{e} + \frac{1}{d}\right) P_{ijB} + \sum_{j=5}^{6} \sum_{i=0}^{5} P_{Li} P_{Nj} \left(\frac{1}{e} + \frac{2}{d}\right) P_{ijB}$$

$$= (.428\text{x}10^{-3})(.1+.1)(5.2)$$
$$+ (.428\text{x}10^{-3})(.1+.1)(5.2)$$
$$+ (.856\text{x}10^{-4})(.1+.1)(5.2)$$
$$+ (.5136\text{x}10^{-3})(.1+.1)(5.2)$$
$$+ (.256\text{x}10^{-6})(1)(8.4)$$
$$= 1.51\text{x}10^{-3} \text{ occ/day} = .552 \text{ occ/yr}$$

PROPORTIONAL IMPEDANCE

$$P_C = (.428\text{x}10^{-3})(.1+.1+.1)$$
$$+ (.428\text{x}10^{-3})(.1+.1+.1)$$
$$+ (.856\text{x}10^{-4})(.1+.1)$$
$$+ (.5136\text{x}10^{-3})(.1+.1)$$
$$+ (.256\text{x}10^{-6})(.1+.1)$$
$$+ (.44\text{x}10^{-7})(1)$$
$$= .377\text{x}10^{-3}$$

$$P_B = (.428\text{x}10^{-3})(.1+.1+.1)(5.2)$$
$$+ (.428\text{x}10^{-3})(.1+.1+.1)(5.2)$$
$$+ (.856\text{x}10^{-4})(.1+.1)(5.2)$$
$$+ (.5136\text{x}10^{-3})(.1+.1)(5.2)$$
$$+ (.256\text{x}10^{-6})(1)(8.4)$$
$$= 1.96\text{x}10^{-3} \text{ occ/day} = .715 \text{ occ/yr}$$

EQUAL IMPEDANCE

$$F_C = (.428\text{x}10^{-3})(.1+.1)(5.2)$$
$$+ (.428\text{x}10^{-3})(.1+.1)(5.2)$$
$$+ (.856\text{x}10^{-4})(.1+.1)(5.2)$$
$$+ (.5136\text{x}10^{-3}(.1+.1)(5.2)$$
$$+ (.256\text{x}10^{-6})(.1+.1)(8.4)$$
$$+ (.44\text{x}10^{-7})(1)(8.4)$$
$$= 1.51\text{x}10^{-3} \text{ occ/day}$$
$$= .552 \text{ occ/yr}$$

PROPORTIONAL IMPEDANCE

$$F_C = (.428\text{x}10^{-3})(.1+.1+.1)(5.2)$$
$$+ (.428\text{x}10^{-3})(.1+.1+.1)(5.2)$$
$$+ (.856\text{x}10^{-4})(.1+.1)(5.2)$$
$$+ (.5136\text{x}10^{-3})(.1+.1)(5.2)$$
$$+ (.256\text{x}10^{-6})(.1+.1)(8.4)$$
$$+ (.44\text{x}10^{-7})(1)(8.4)$$
$$= 1.96\text{x}10^{-3} \text{ occ/day}$$
$$= .715 \text{ occ/yr}$$

For 100 MW Line Capacity

$$P_B = (.256\text{x}10^{-6})(1)$$
$$= .256\text{x}10^{-6}$$

$$P_B = (.428\text{x}10^{-3})(.1)$$
$$+ (.428\text{x}10^{-3})(.1)$$
$$+ (.256\text{x}10^{-6})(1)$$
$$= .86\text{x}10^{-4}$$

EQUAL IMPEDANCE

$$P_C = (.44\times10^{-7})(1)$$
$$= .44\times10^{-7}$$

$$F_B = (.256\times10^{-6})(1)(8.4)$$
$$= 2.15\times10^{-6} \text{ occ/day}$$
$$= .784\times10^{-3} \text{ occ/yr}$$

$$F_C = (.44\times10^{-7})(1)(8.4)$$
$$= .370\times10^{-6} \text{ occ/day}$$
$$= .135\times10^{-3} \text{ occ/yr}$$

PROPORTIONAL IMPEDANCE

$$P_C = (.428\times10^{-3})(.1)$$
$$+ (.428\times10^{-3})(.1)$$
$$+ (.44\times10^{-7})(1)$$
$$= .86\times10^{-4}$$

$$F_B = (.428\times10^{-3})(1)(5.2)$$
$$+ (.428\times10^{-3})(.1)(5.2)$$
$$+ (.256\times10^{-6})(1)(8.4)$$
$$= .445\times10^{-3} \text{ occ/day}$$
$$= .162 \text{ occ/yr}$$

$$F_C = (.428\times10^{-3})(.1)(5.2)$$
$$+ (.428\times10^{-3})(.1)(5.2)$$
$$+ (.44\times10^{-7})(1)(8.4)$$
$$= .445\times10^{-3} \text{ occ/day}$$
$$= .162 \text{ occ/yr}$$

The variance of these indices can be calculated using the equations derived in Chapter IV. If the line failure probability for length (P_ℓ) is stochastic, it can be described by its mean μ_ℓ and variance σ_ℓ^2. The bus outage statistics are μ_{BF}, σ_{BF}^2, μ_{CF}, σ_{CF}^2. For the equal line impedance case

$$P_{B(80)} = (.291\text{x}10^{-3})P_\ell + P_{BF}$$

$$\sigma^2_{B(80)} = (.291\text{x}10^{-3})^2\sigma_\ell^2 + \sigma^2_{BF}$$

$$P_{C(80)} = (.291\text{x}10^{-3})P_\ell + P_{CF}$$

$$\sigma^2_{C(80)} = (.291\text{x}10^{-3})^2\sigma_\ell^2 + \sigma^2_{CF}$$

$$P_{B(100)} = (.256\text{x}10^{-6})P_\ell + P_{BF}$$

$$\sigma^2_{B(100)} = (.256\text{x}10^{-6})\sigma_\ell^2 + \sigma^2$$

$$P_{C(100)} = (.44\text{x}10^{-7})P_\ell + P_{CF}$$

$$\sigma^2_{C(100)} = (.44\text{x}10^{-7})^2P_\ell + P_{CF}$$

The variance for the other indices can be calculated in a similar manner.

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